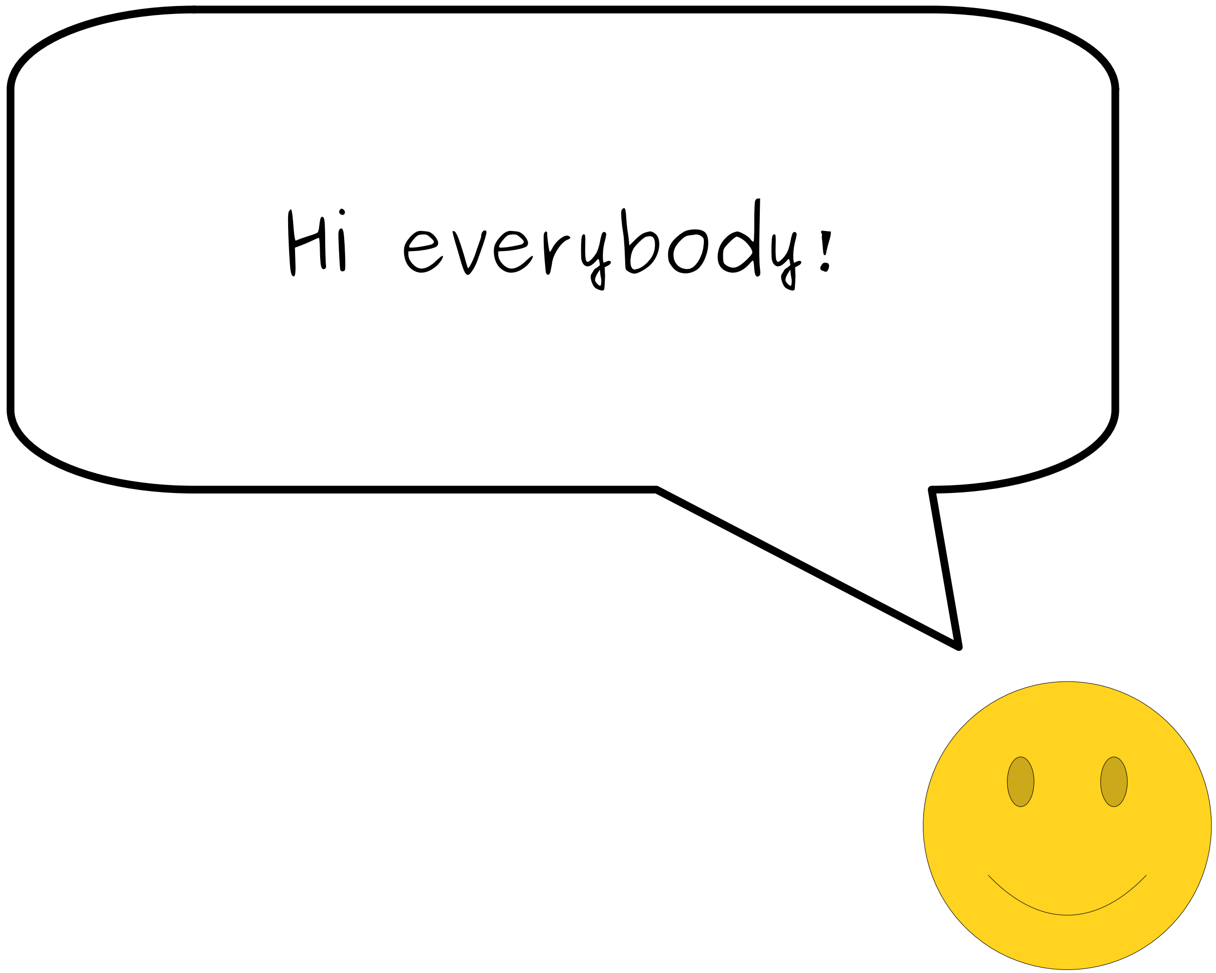
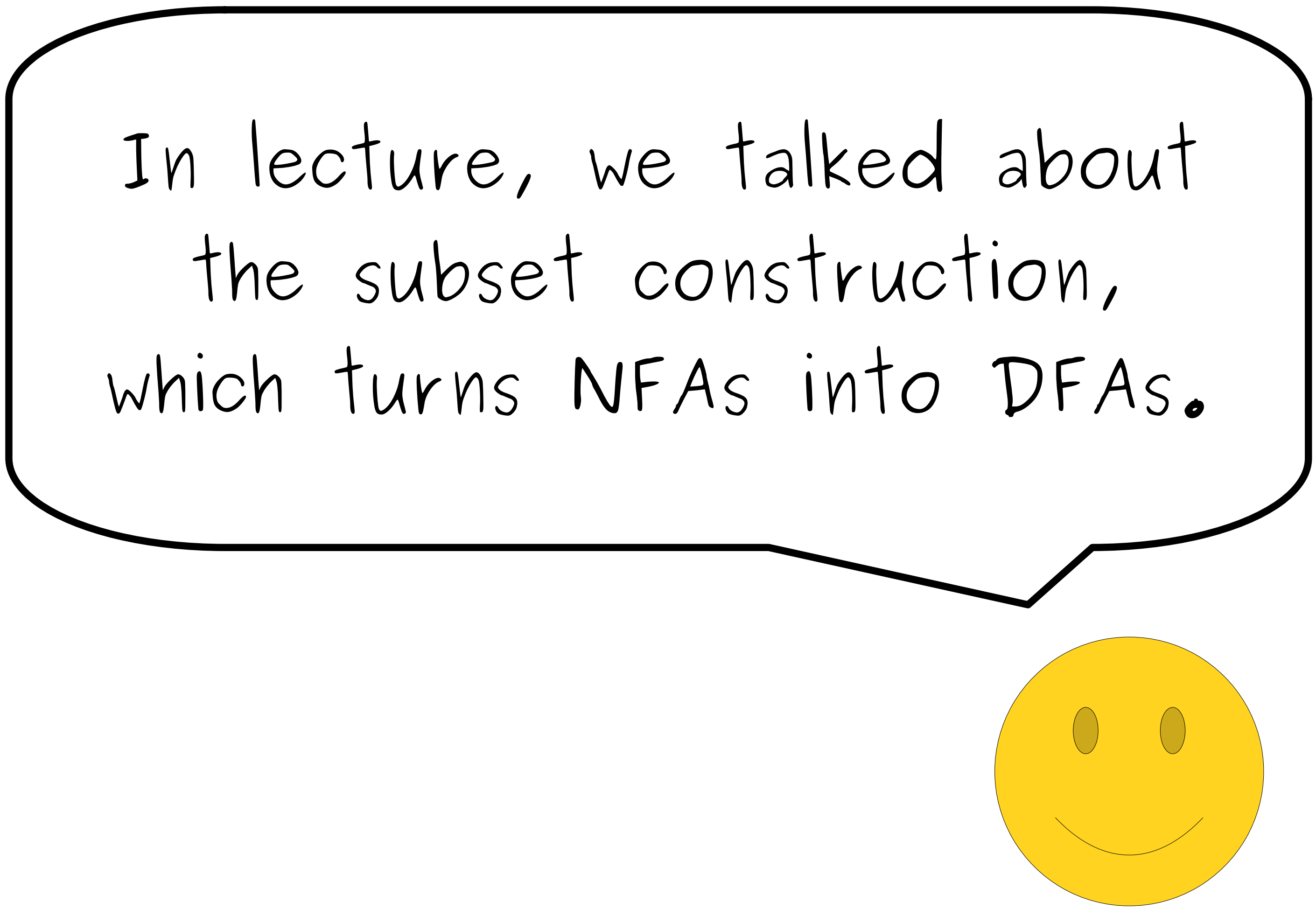
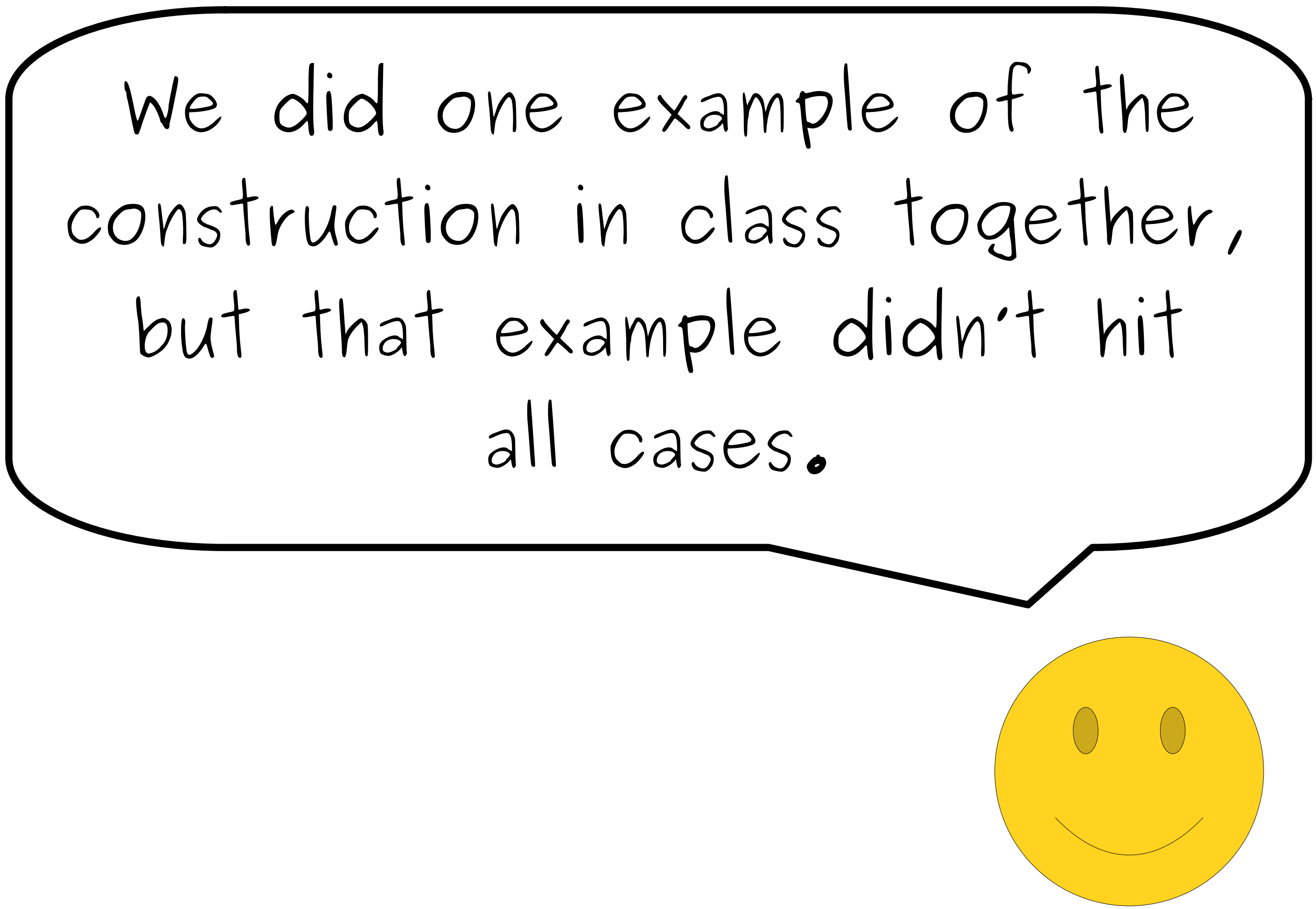
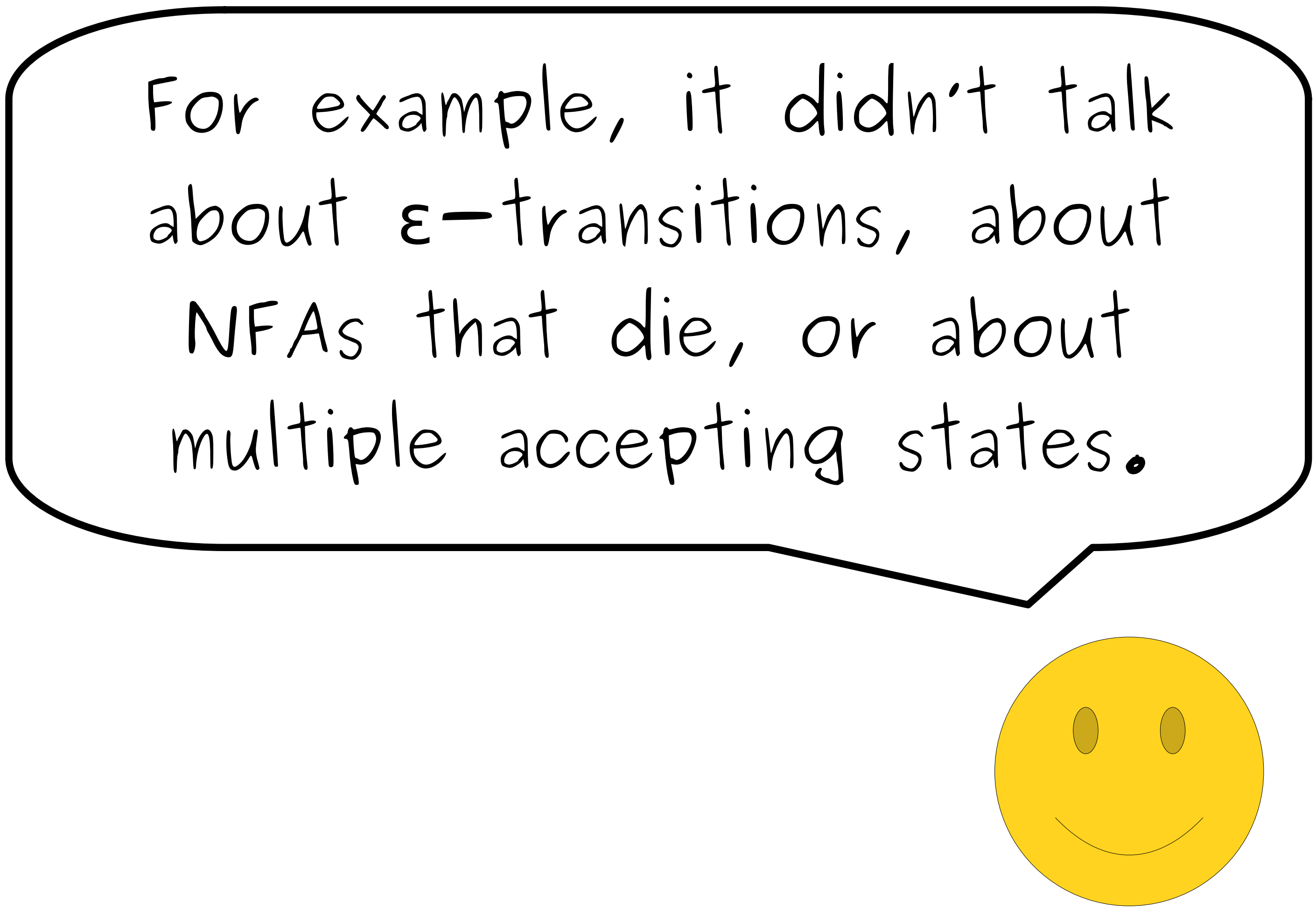
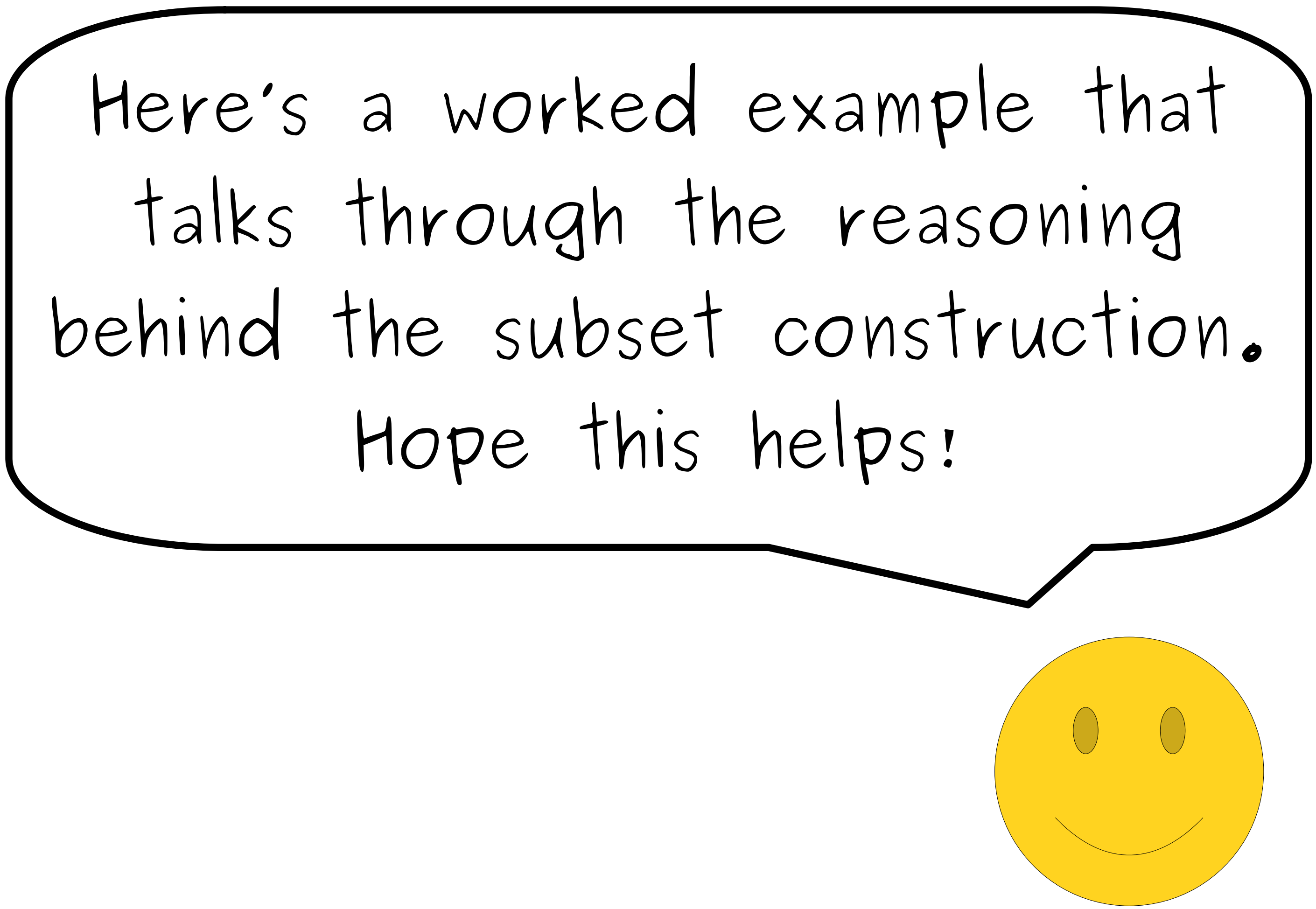
Guide to the Subset Construction


Hi everybody!


In lecture, we talked about the subset construction, which turns NFAs into DFAs.


We did one example of the construction in class together, but that example didn't hit all cases.

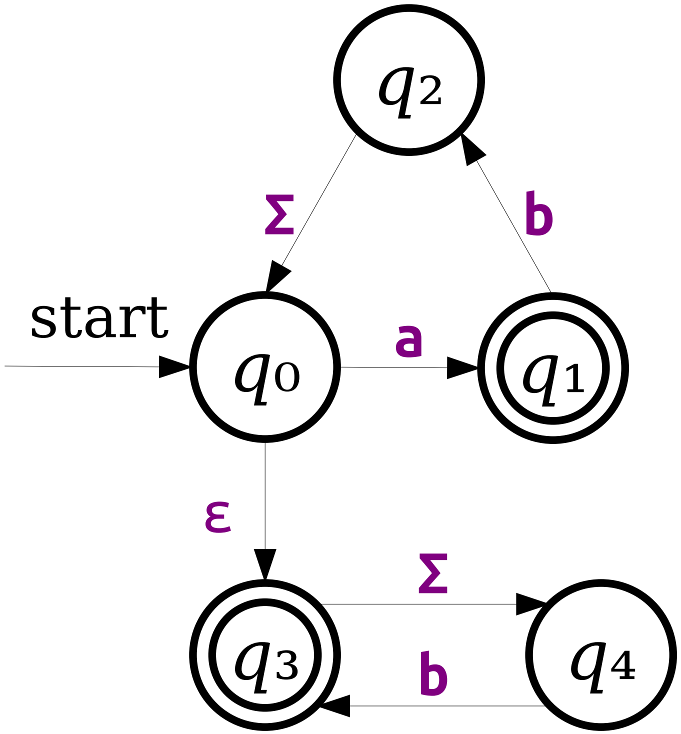

For example, it didn't talk about ε-transitions, about NFAs that die, or about multiple accepting states.


Here's a worked example that talks through the reasoning behind the subset construction. Hope this helps!

Because there's going to be a lot of content on these slides, I'm going to change how I look from the normal "Guide to X" setup.

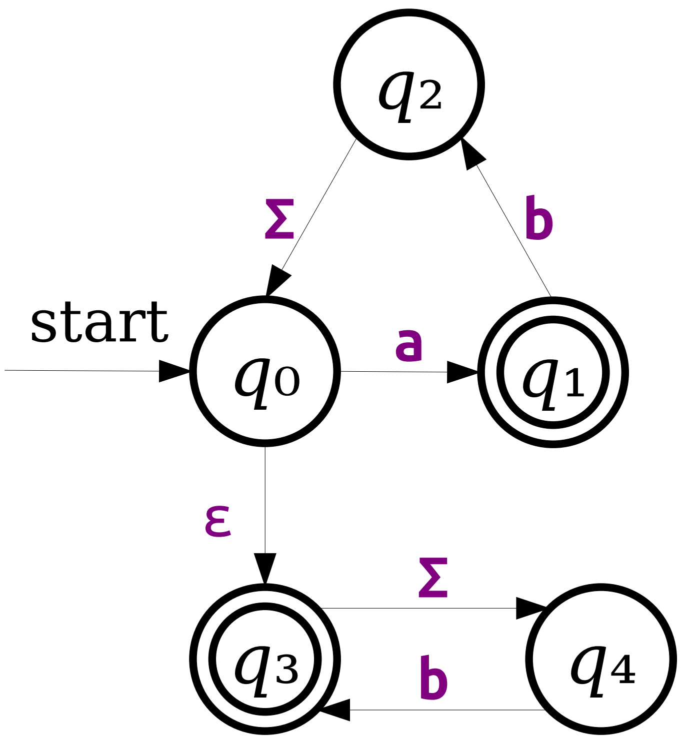
I know that I now look like a super formal piece of text captioning something, but I promise that this is still me and that I'm acting as the narrator for this section.

Once More, With Epsilons!



Here's the NFA that we're going to convert into a DFA. (Or rather, we're going to build a new automaton that's a DFA that has the same language as this NFA. But you can think of it as performing some sort of conversion if you'd like.)

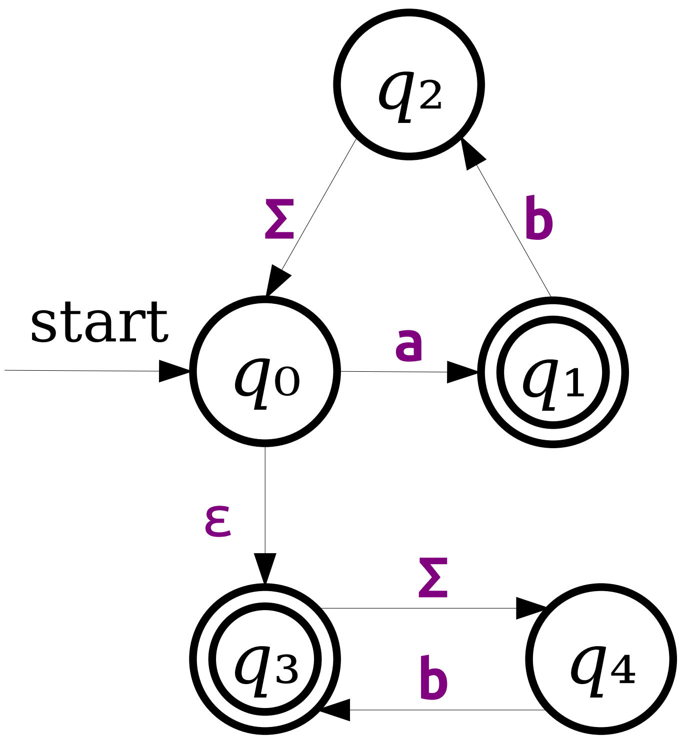
Once More, With Epsilons!



	a	b

We'll represent this new DFA as a table, since that's usually *much* easier than drawing a state transition diagram!

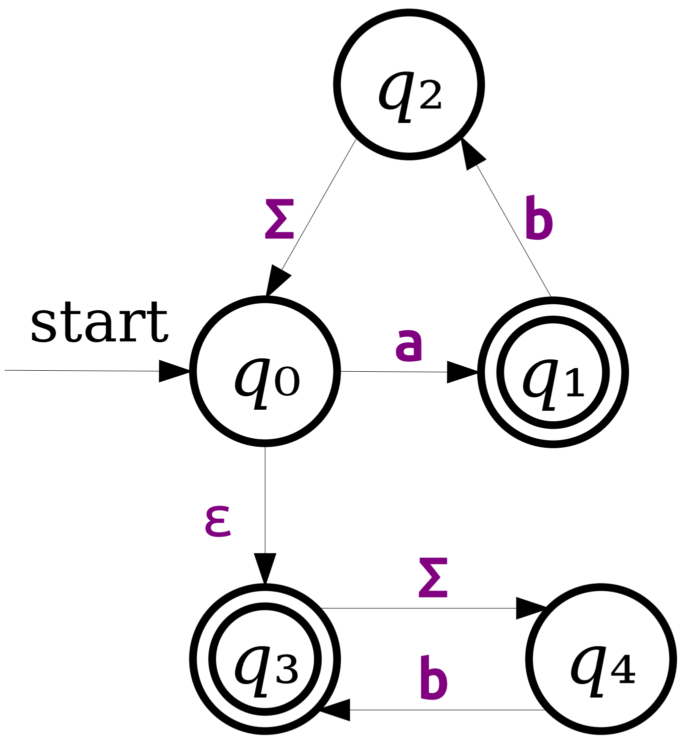
Once More, With Epsilons!



	a	b

There are more rows here than states, and that’s normal. The subset construction usually makes a DFA with more states than the NFA.

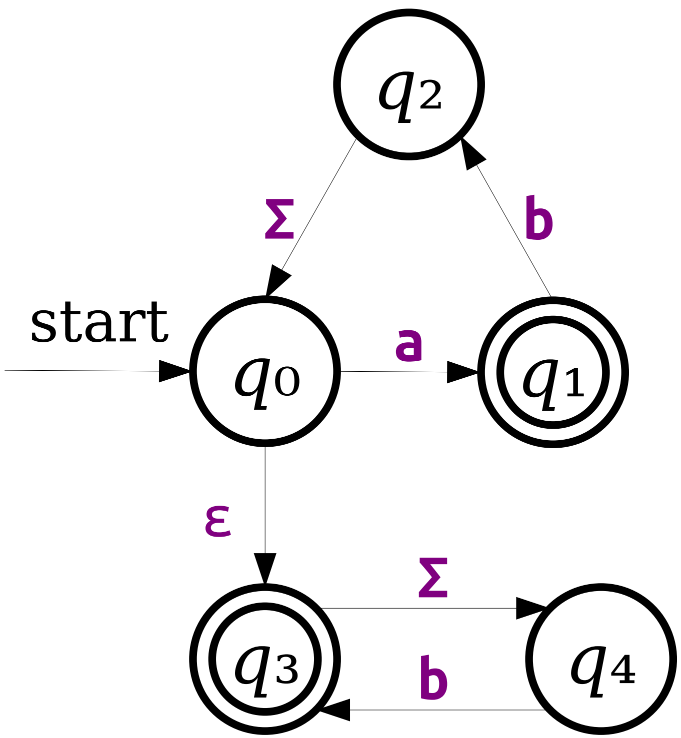
Once More, With Epsilons!



	a	b

There's no general way to predict how many rows you'll need. We've scouted ahead here and made this table exactly the right size.

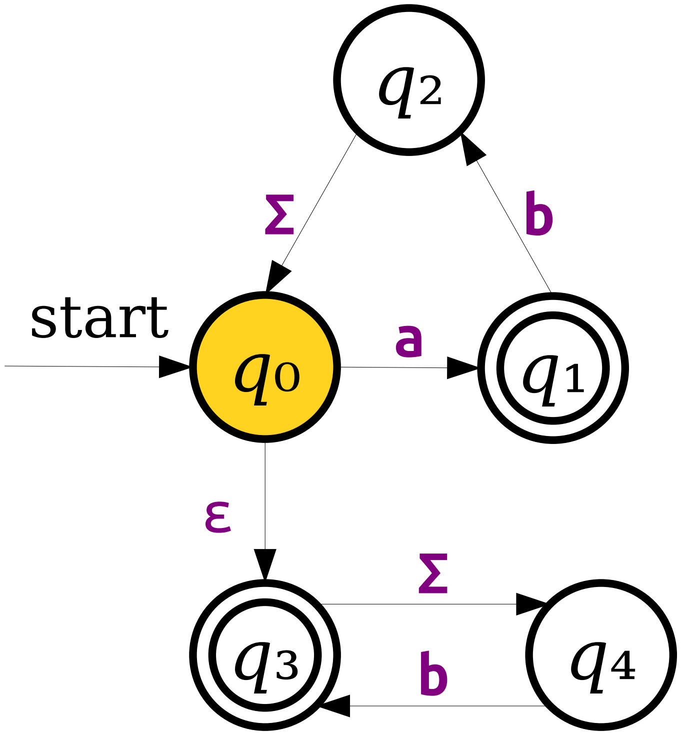
Once More, With Epsilons!



	a	b

Let's start off by thinking about what the start state of our DFA is going to be.

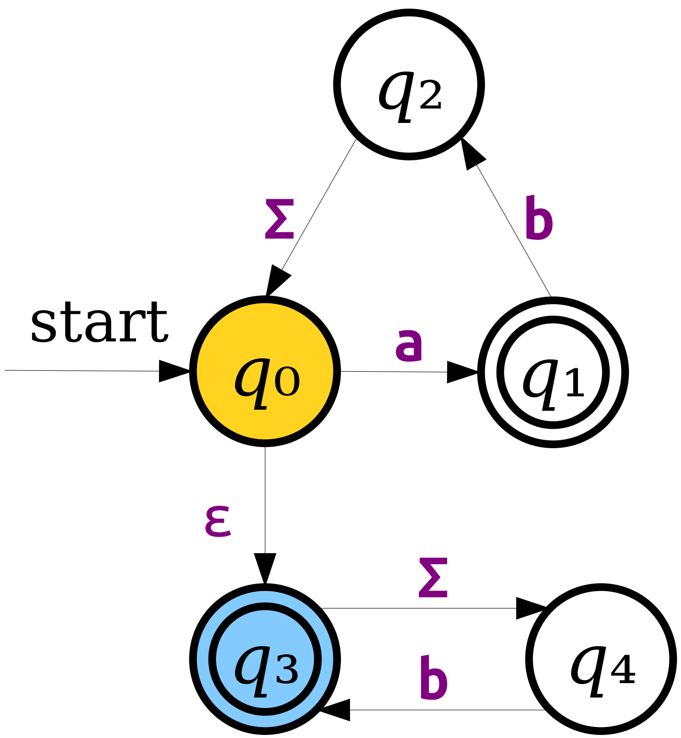
Once More, With Epsilons!



	a	b

To do that, let's see what happens when we power on the NFA. We'll begin in the start state, as usual.

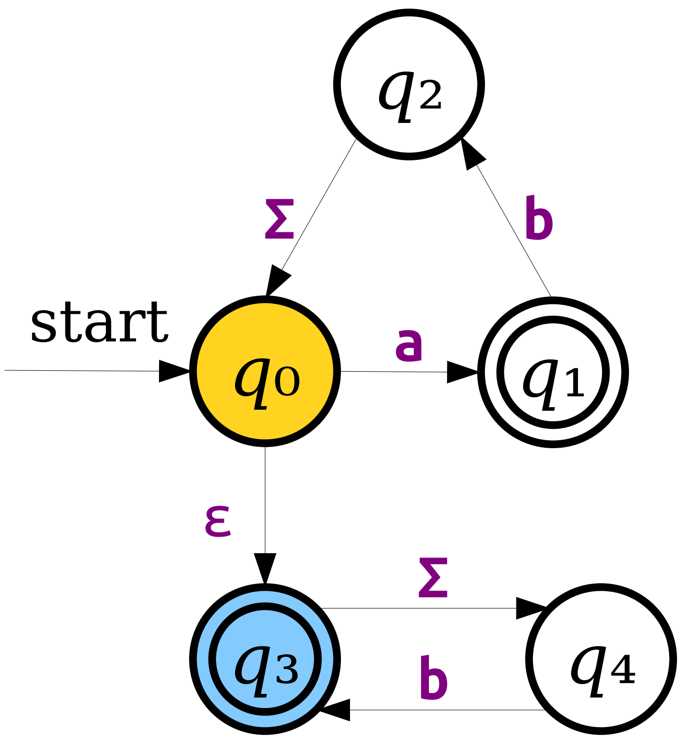
Once More, With Epsilons!



	a	b

But… there’s a twist. When this NFA powers on, if we use the “massive parallel” intuition, we’d also end up in state q_3 because there’s an ε-transition from state q_0 to state q_3.

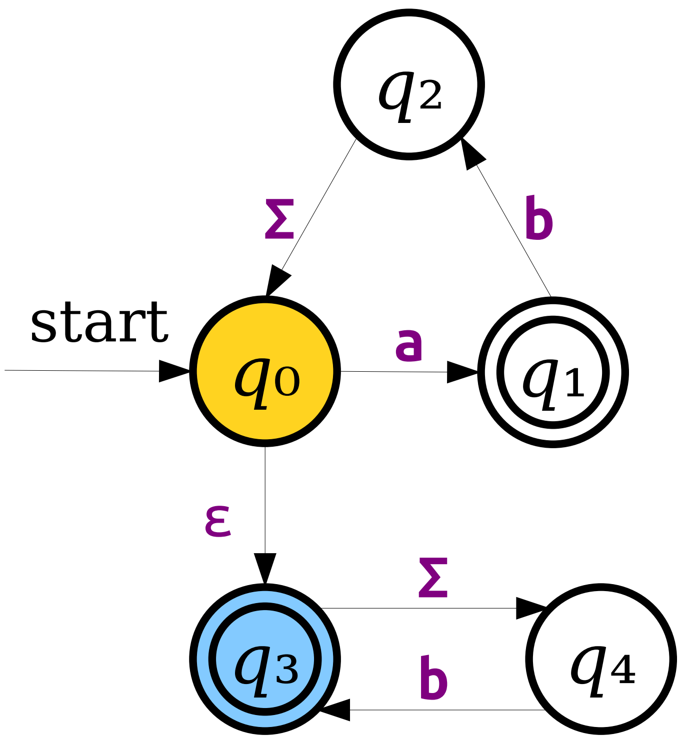
Once More, With Epsilons!



	a	b

Note that we ***don't*** include state q_4 here. The transition from q_3 to q_4 is a Σ-transition rather than an ε-transition. Remember that Σ means "you can take this transition when reading any character," whereas an ε-transition means "you can take this transition whenever you'd like."

Once More, With Epsilons!

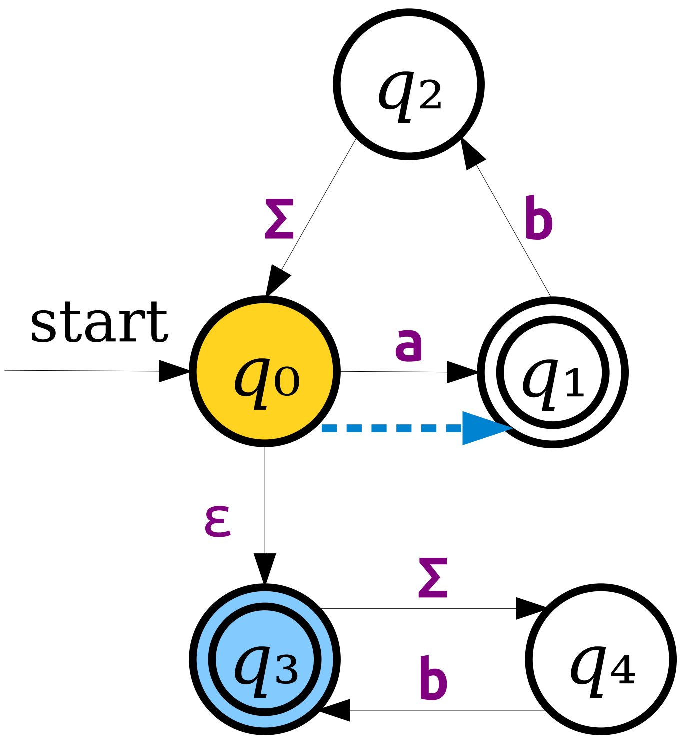


	a	b
$\{q_0, q_3\}$		

Now, we ask – what would this NFA do in this collection of states if it read a particular character?

Let’s start off by seeing what the NFA would do on an `a`.

Once More, With Epsilons!

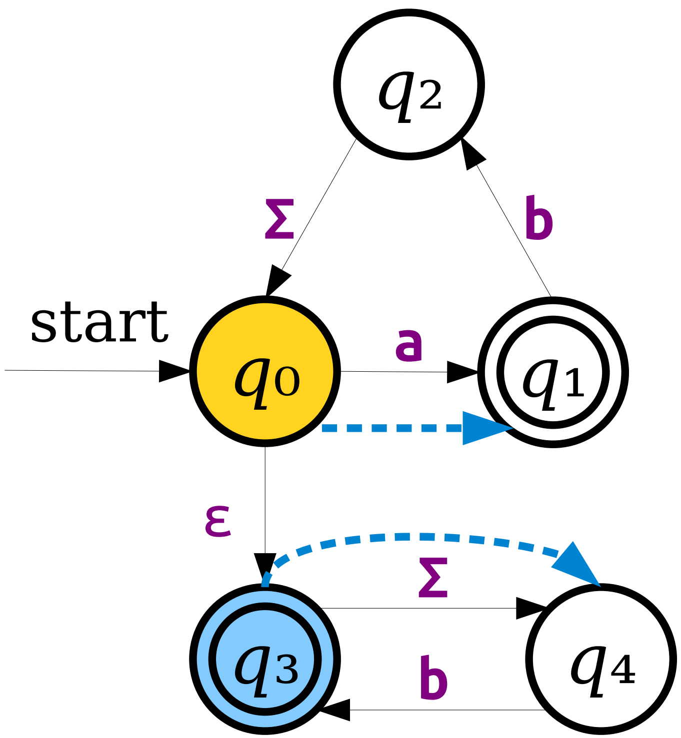


	a	b
$\{q_0, q_3\}$		

State q_0 only has one option – go to state q_1.

We could alternatively take the ε-transition to state q_3 and then try to follow a transition there, but we don't need to consider that here. We're already going to be looking at state q_3 in a second anyway, so following the ε transition here first would be redundant.

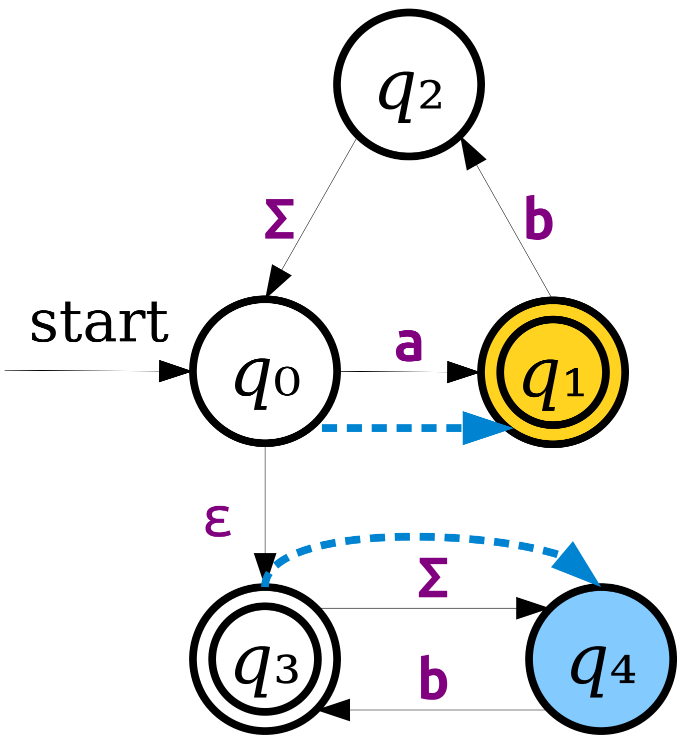
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$		

State q_3 can take its Σ-transition to state q_4. That's another option.

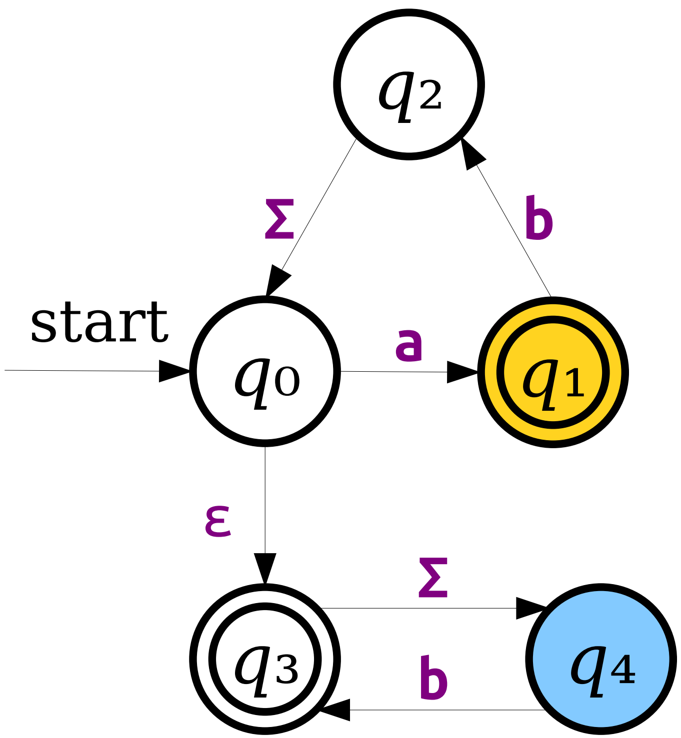
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$		

So here's where we'd end up if we saw an a in these combination of states.

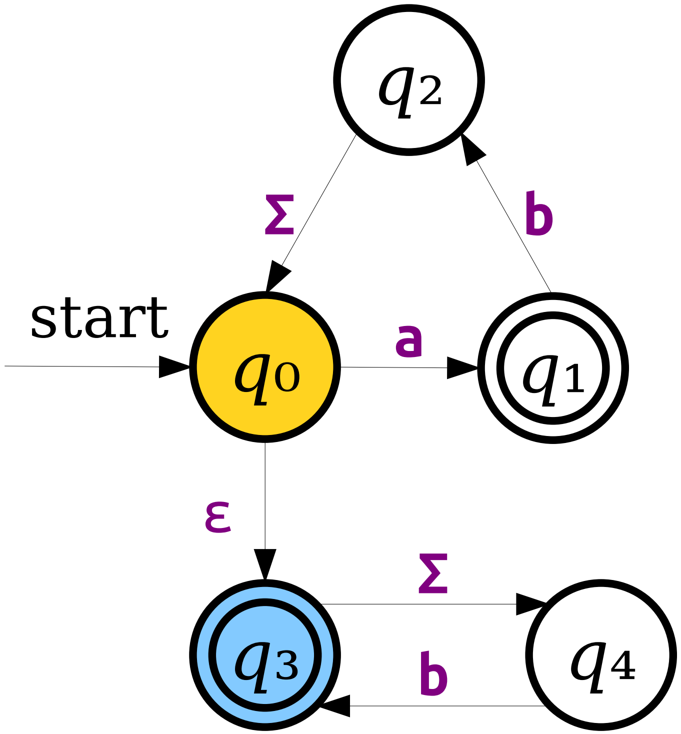
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

So we'll write down this combo of states in the column for a.

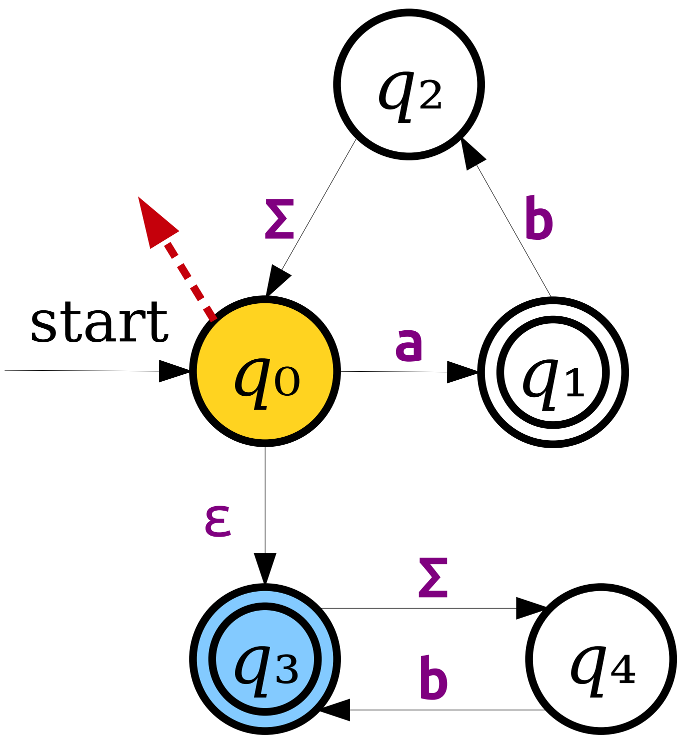
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

Now, let's do the same thing for the character b.

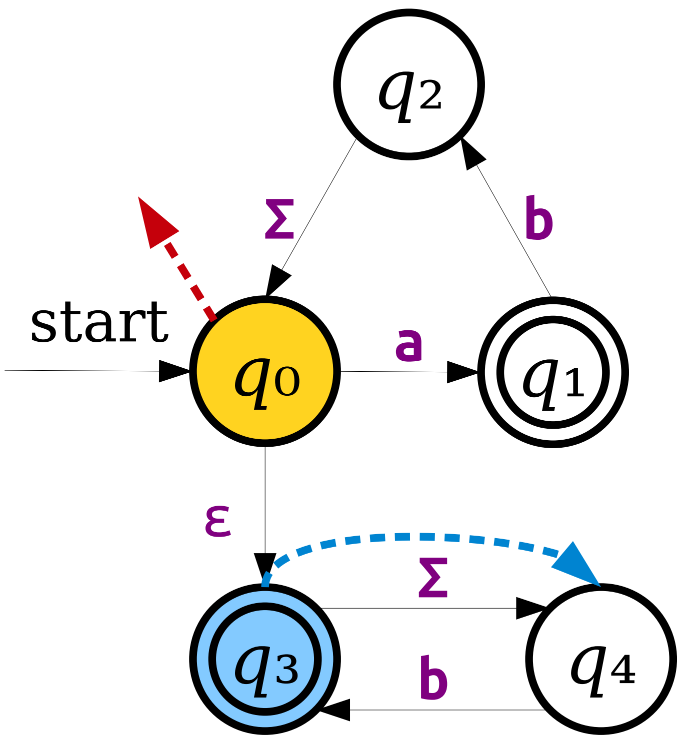
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

State q_0 has no transitions on b, so that state will die off. (We denote this by having an arrow pointing into the abyss.)

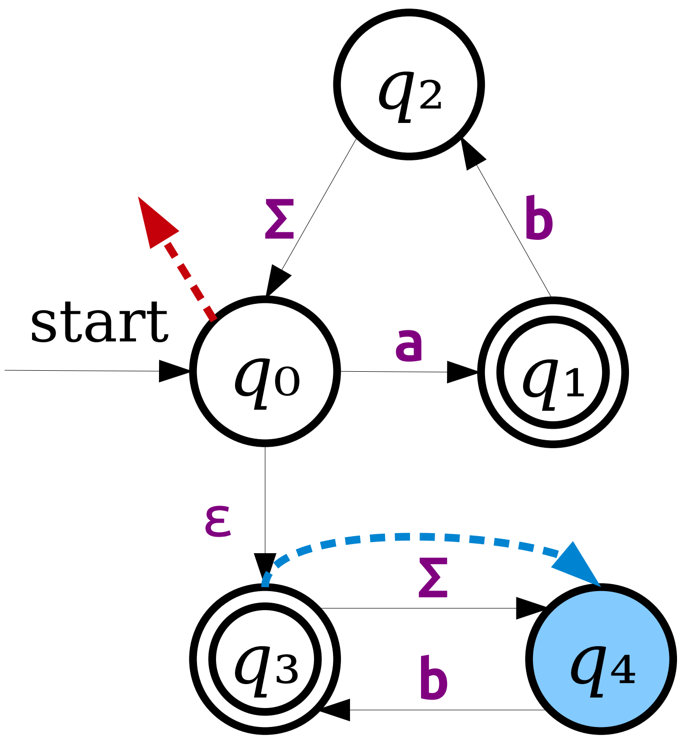
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

State q_3 can only go to q_4.

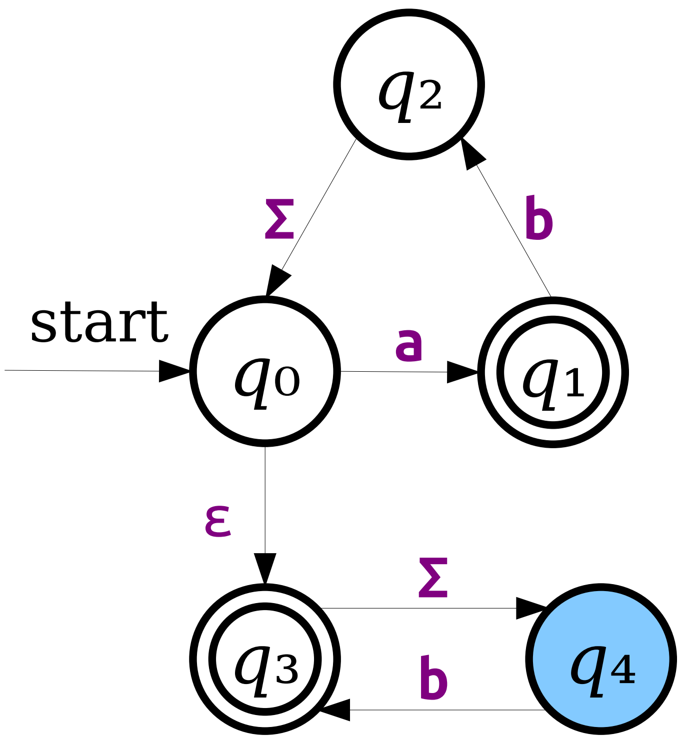
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	

So overall we're in a combination of just one state…

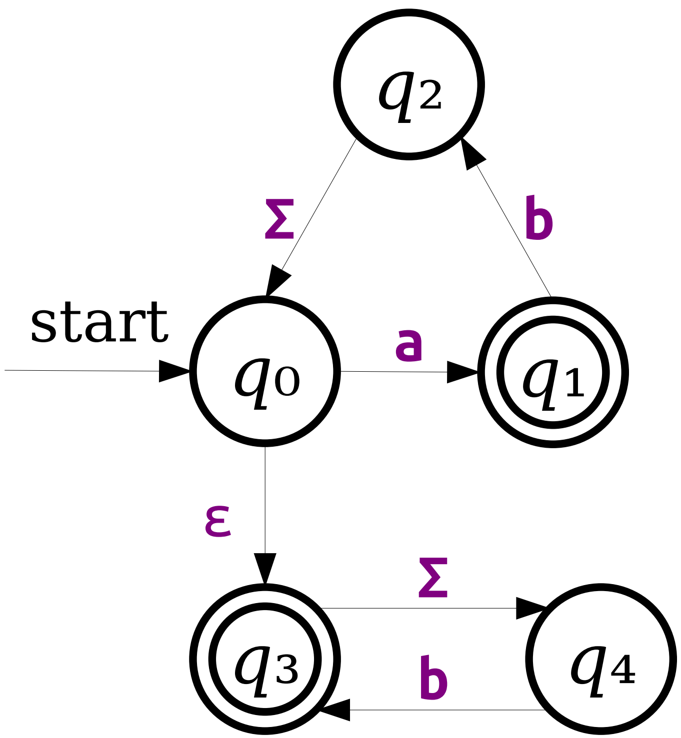
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

… so that's what we'll put into our table.

Once More, With Epsilons!

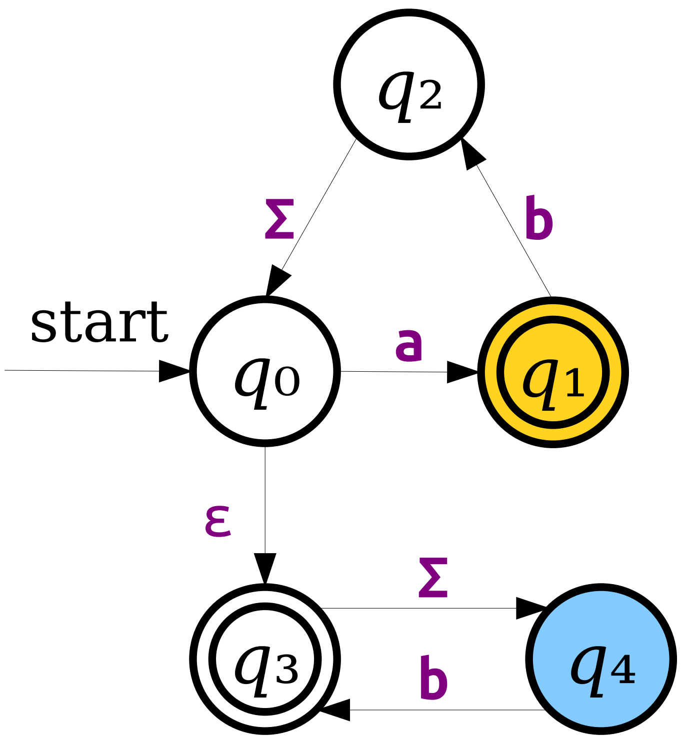


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$

At this point, we're done processing that combination of states. Great!

From this point forward, we'll repeat a simple process: pick a combination of states that's an entry in the table but not yet a row in the table, then apply this same process.

Once More, With Epsilons!

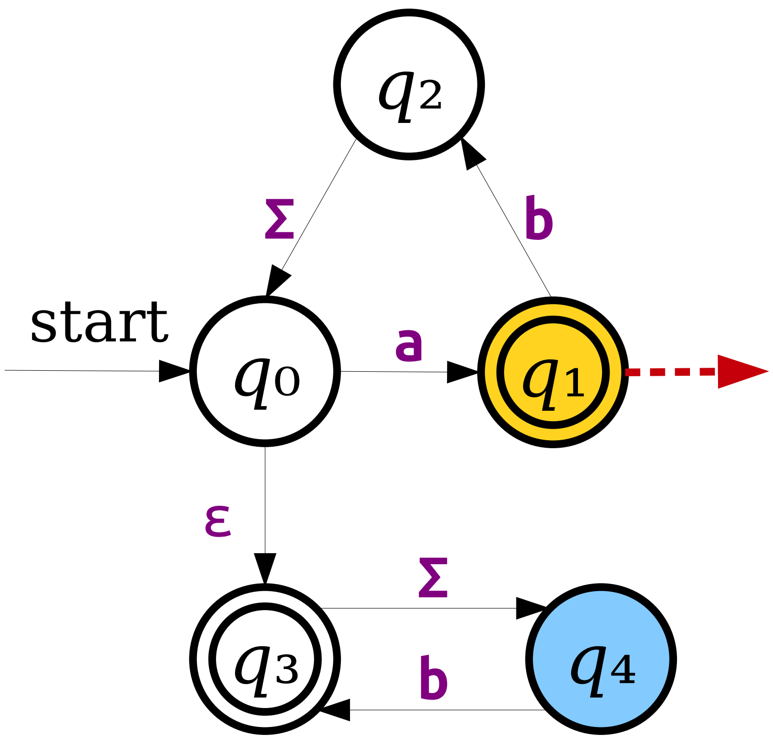


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Let's try this combination.

What happens if we read a?

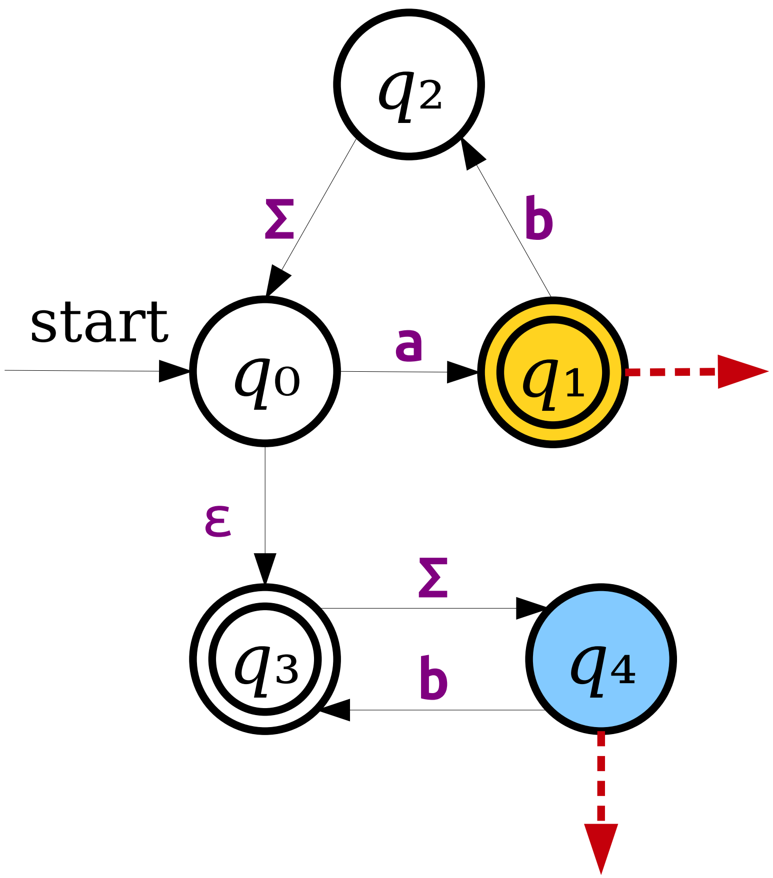
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

Well, state q_1 has nowhere to go, so it'll die off.

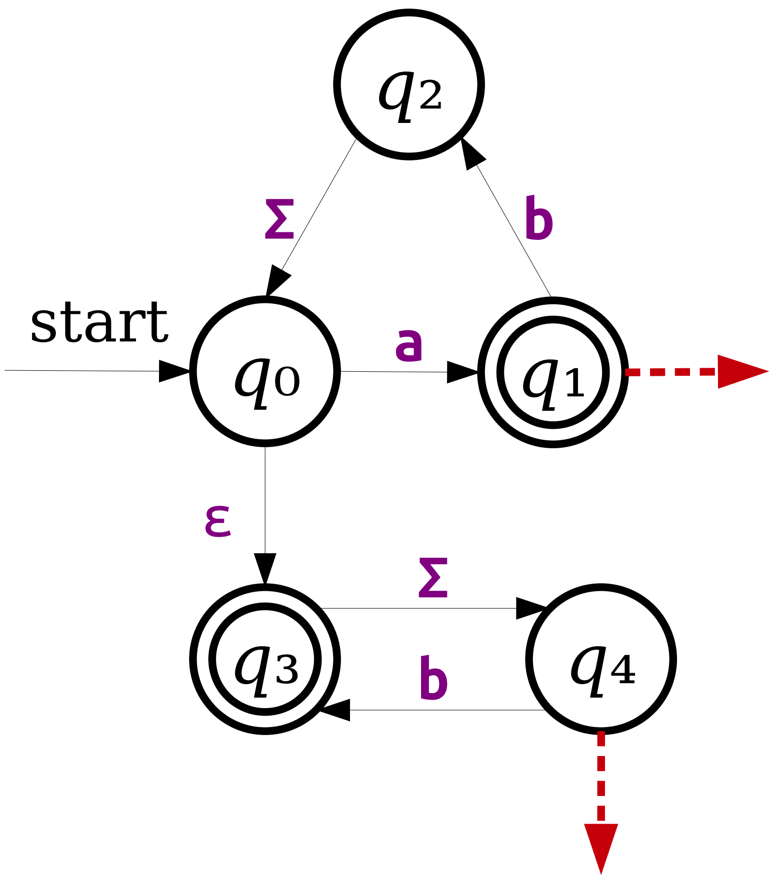
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

And similarly, state q_4 has nowhere to go, so it'll die off as well.

Once More, With Epsilons!

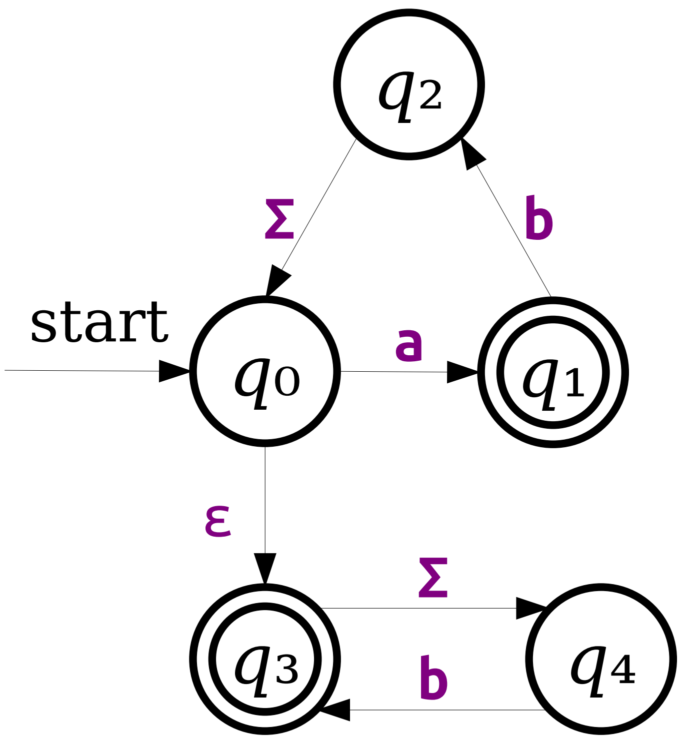


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

That means that we're in this combination of states – nothing is active, since everything died off. (Uh oh).

So what goes in the table here? Make a guess, then move on to the next slide.

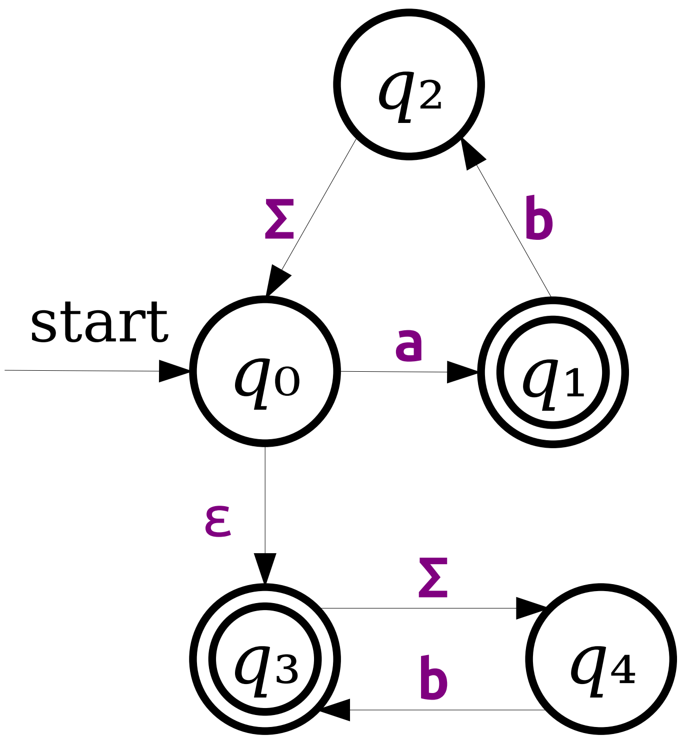
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$		

You've got your guess, right? Because *of course* you wouldn't peek ahead without doing that.

Once More, With Epsilons!

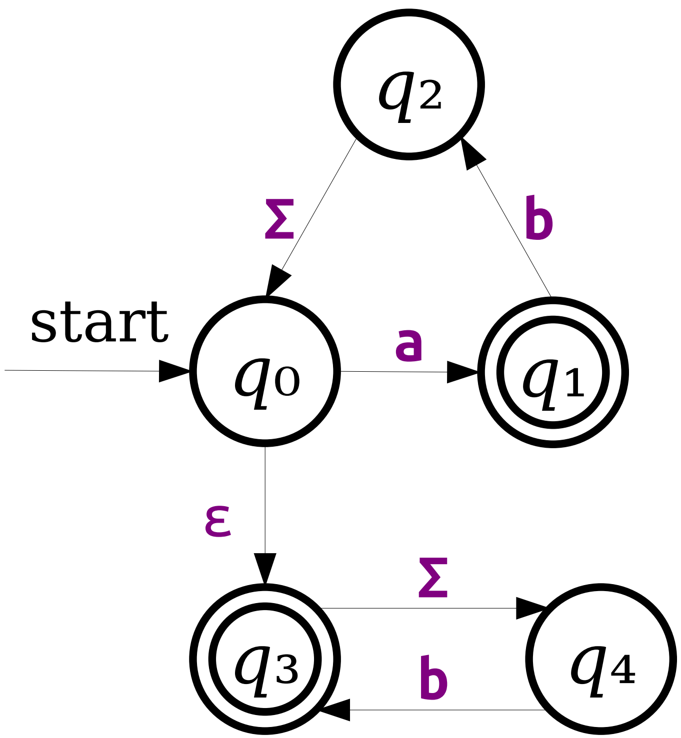


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

We're going to put the empty set here. Why?

The general rule is that we take the set of states that we'd end up in, then put that in the table. Stated differently, we'd gather all the active states into a set and write that set down. There are no active states, so the set of active states is Ø.

Once More, With Epsilons!

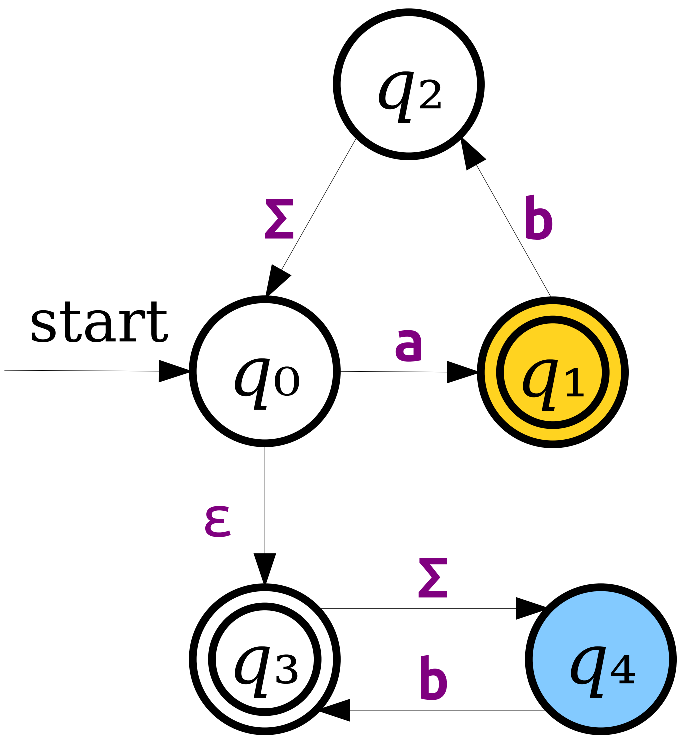


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

More generally, if the NFA ever dies off on some branch, the entry for the table will be the empty set, indicating "we are in a collection of zero NFA states right now."

Remember: DFAs have to have a transition defined for each state/symbol pair. So we have to go somewhere.

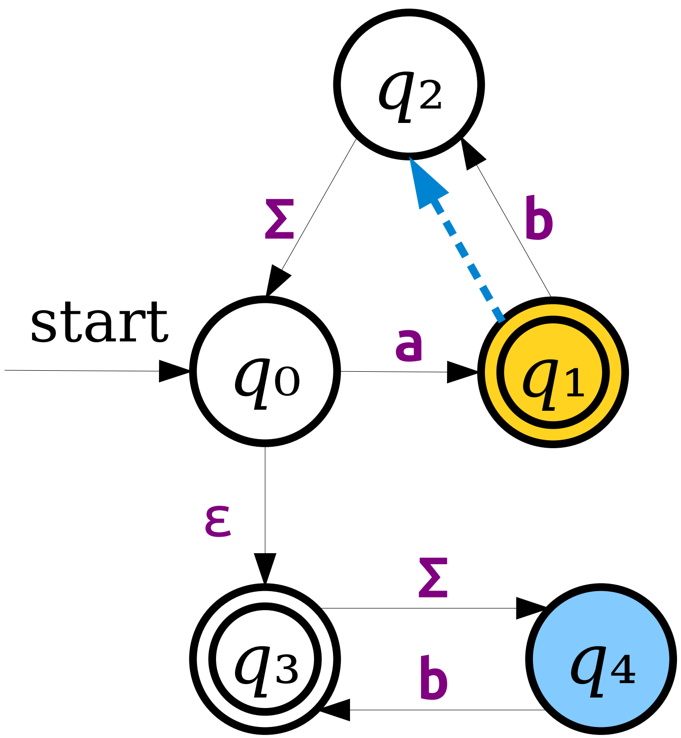
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	

Let’s talk about something happier. What happens if we read b here?

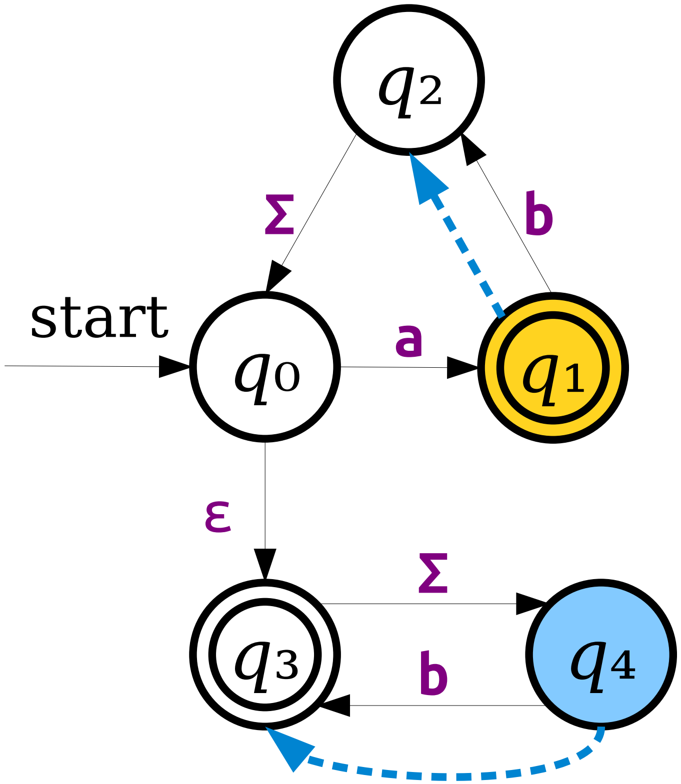
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

State q_1 goes to q_2…

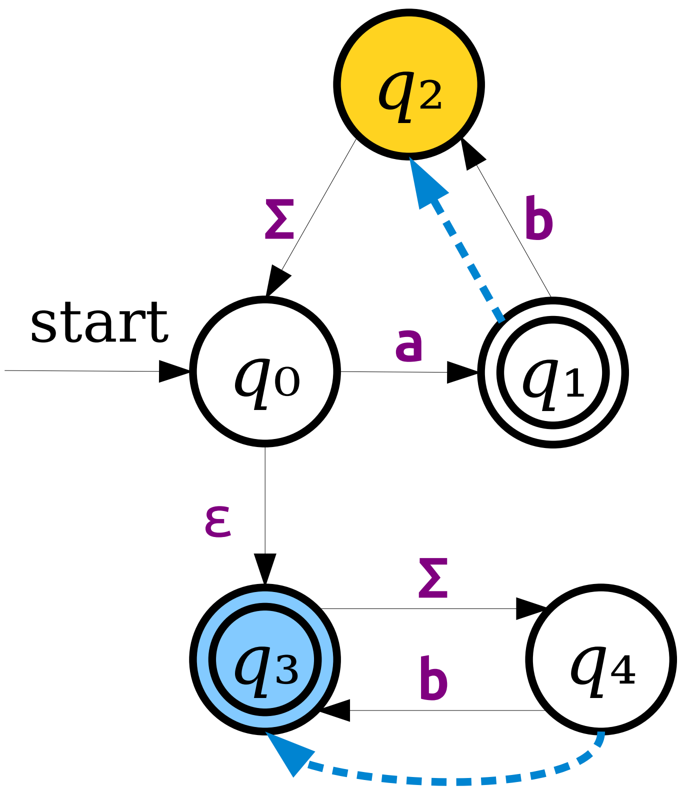
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

… and state q_4 goes to state q_3.

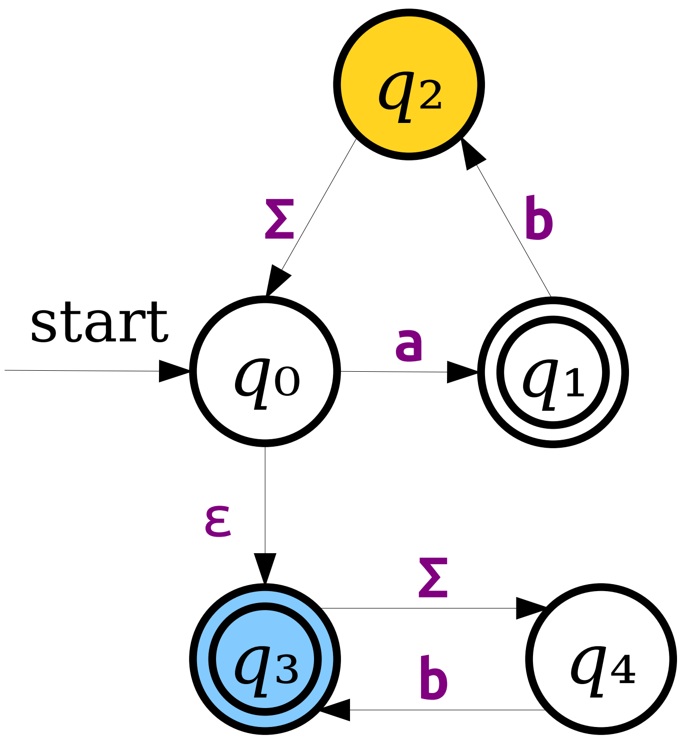
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	

So overall we're now here…

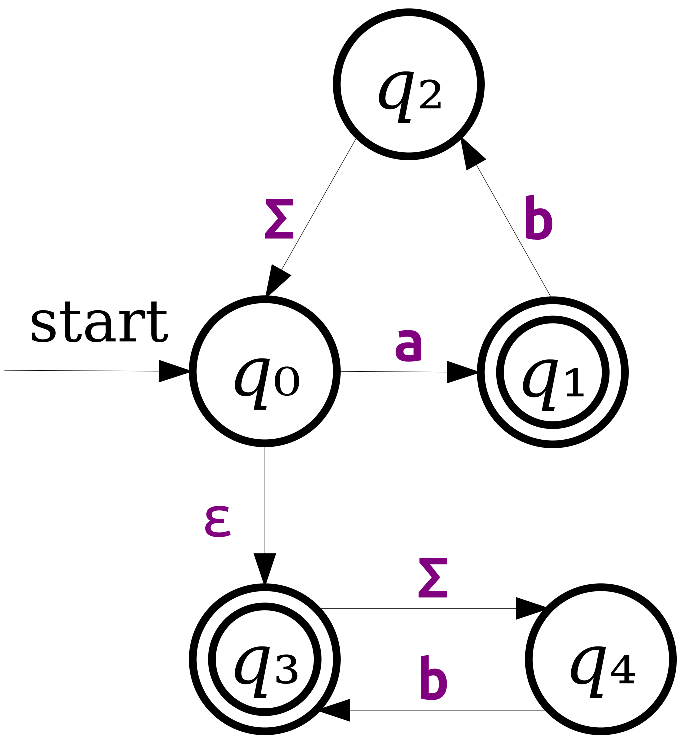
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$

So let's write this combination down.

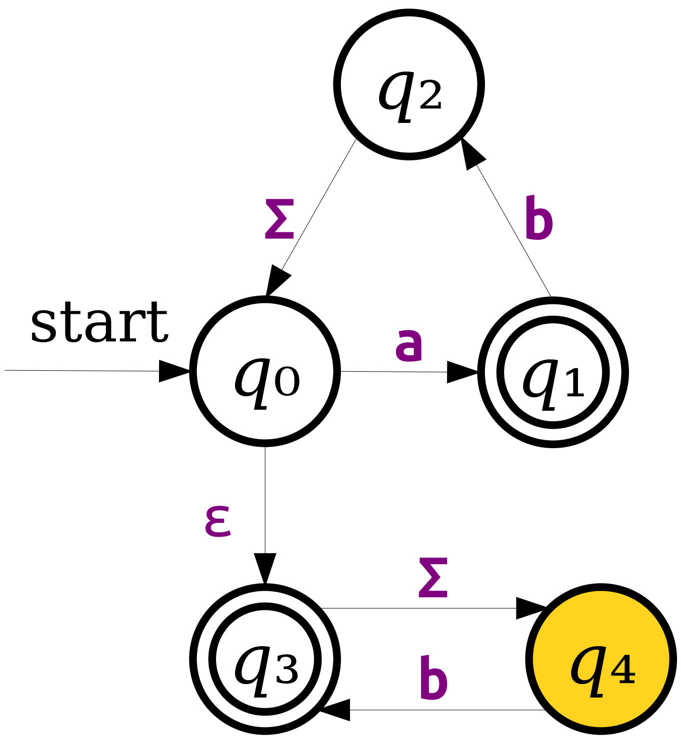
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$

Another row down. Wonderful! Let's pick a new set of states to explore.

Once More, With Epsilons!

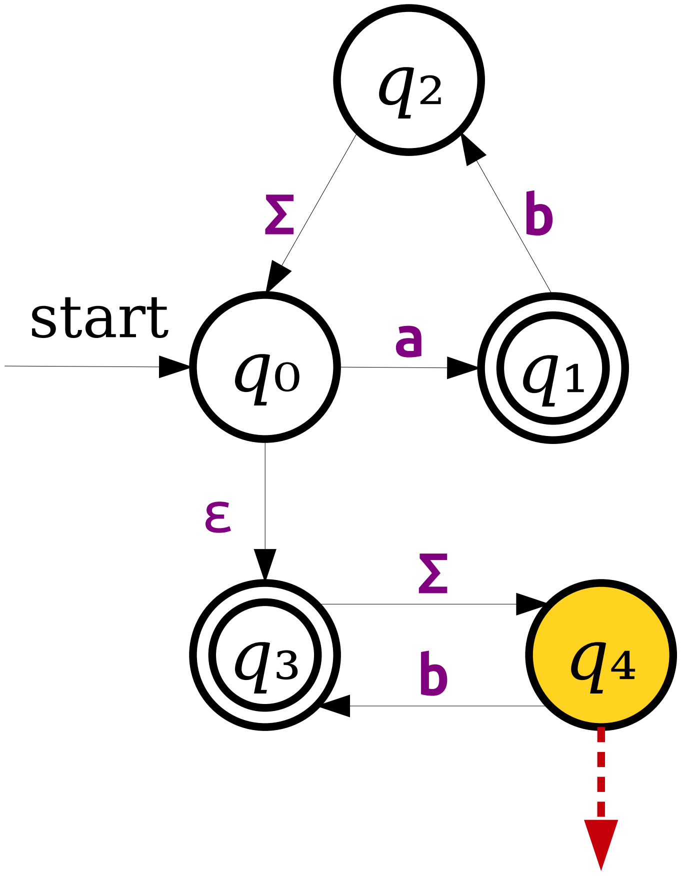


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$		

Here's a nice one. What are the entries in this row going to be?

First, what happens on a?

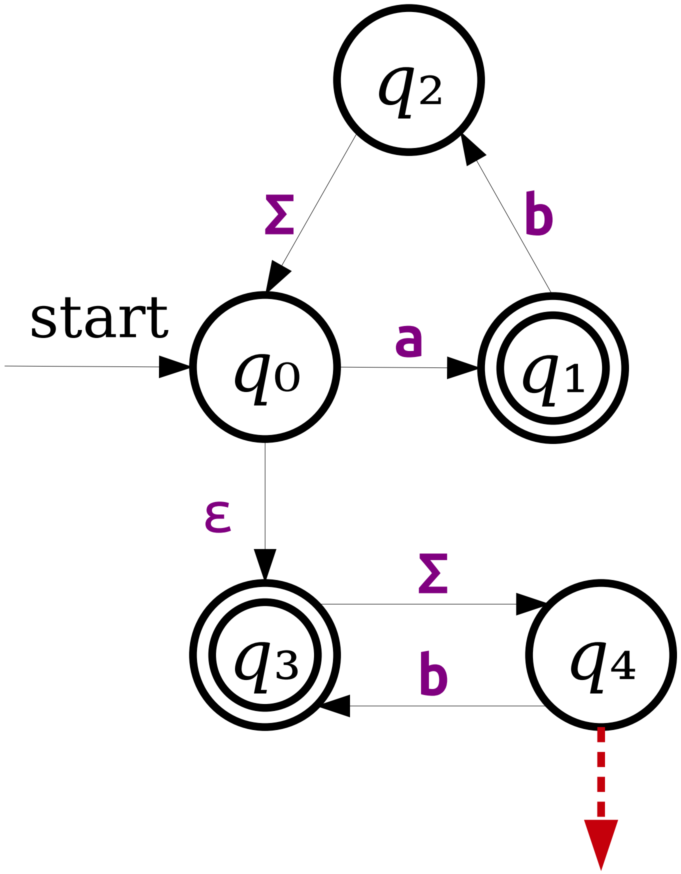
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$		

Well, q_4 dies on a…

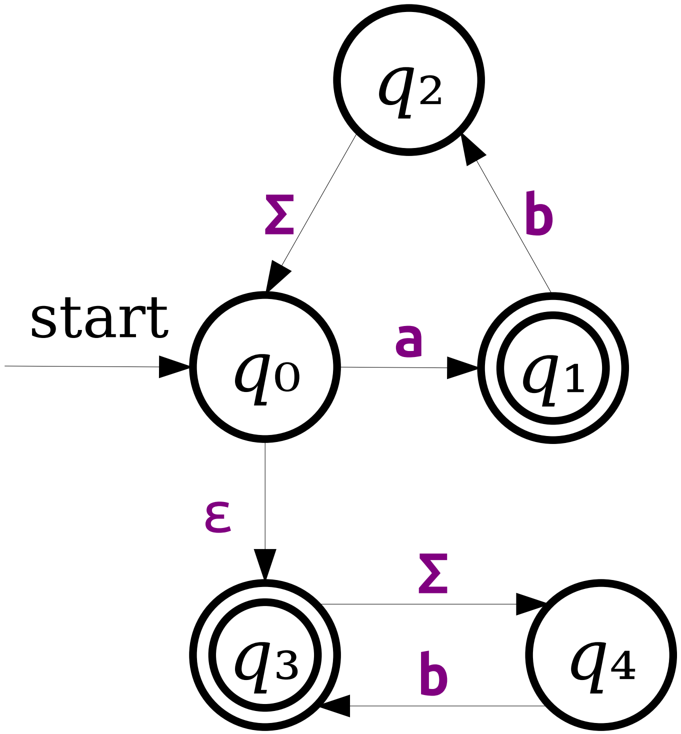
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$		

… so we end up in no states …

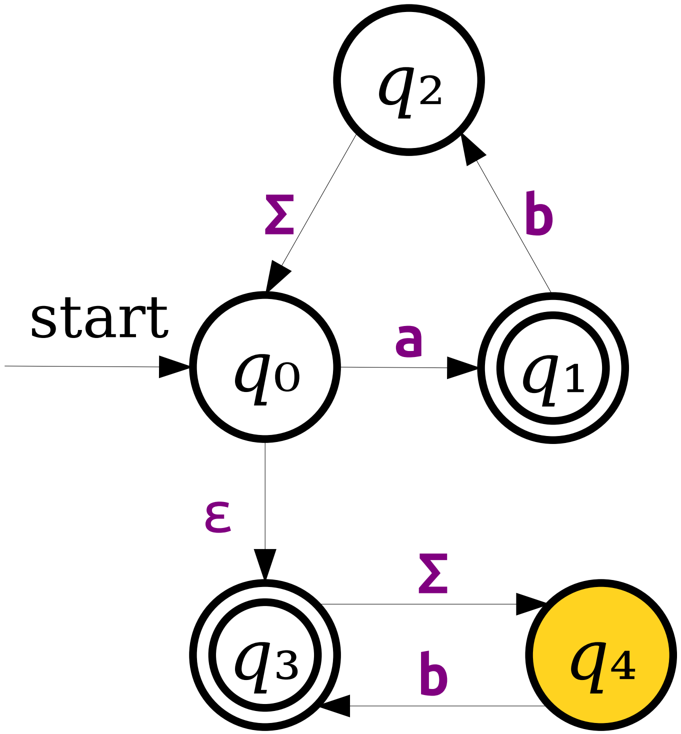
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	

So we write down the empty set here. Done.

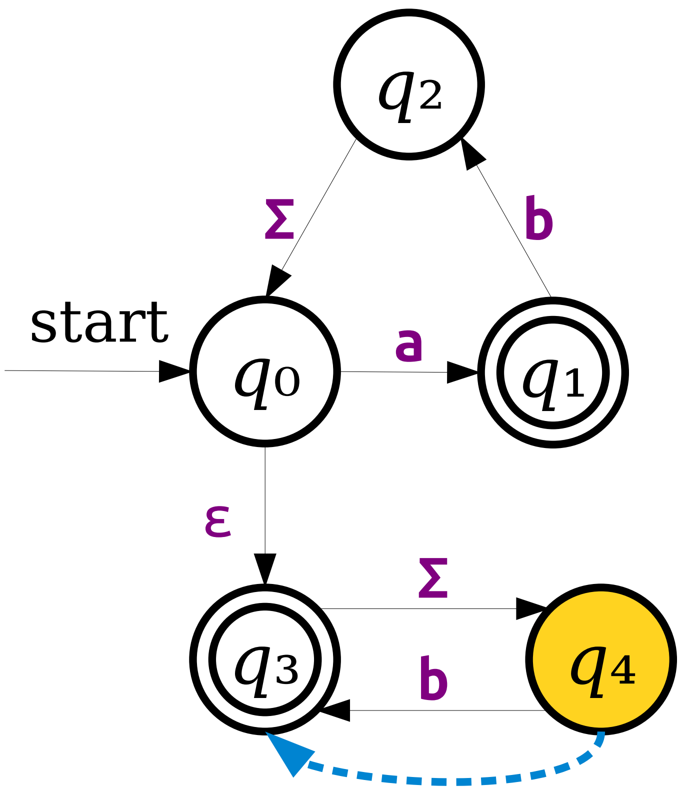
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

What if we read b?

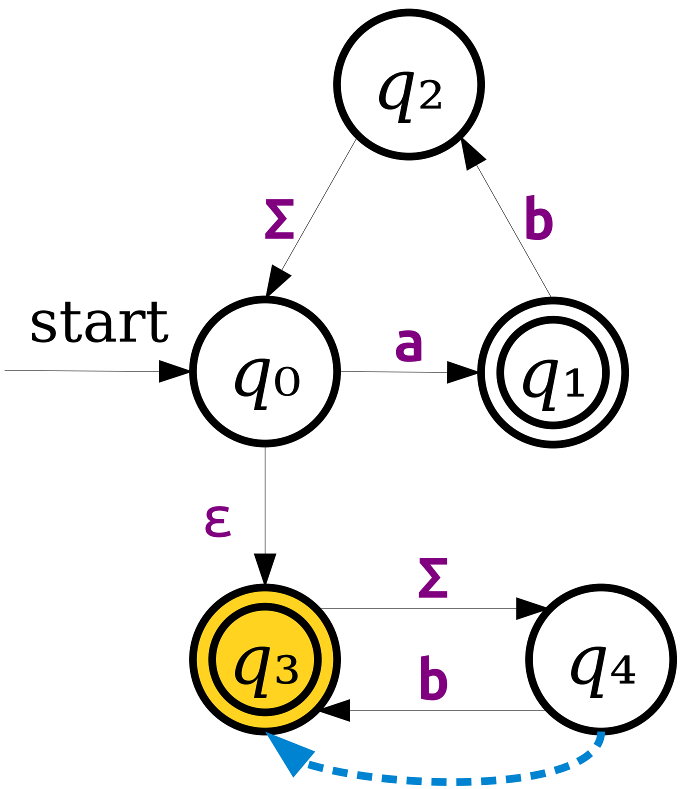
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

That would take us to q_3...

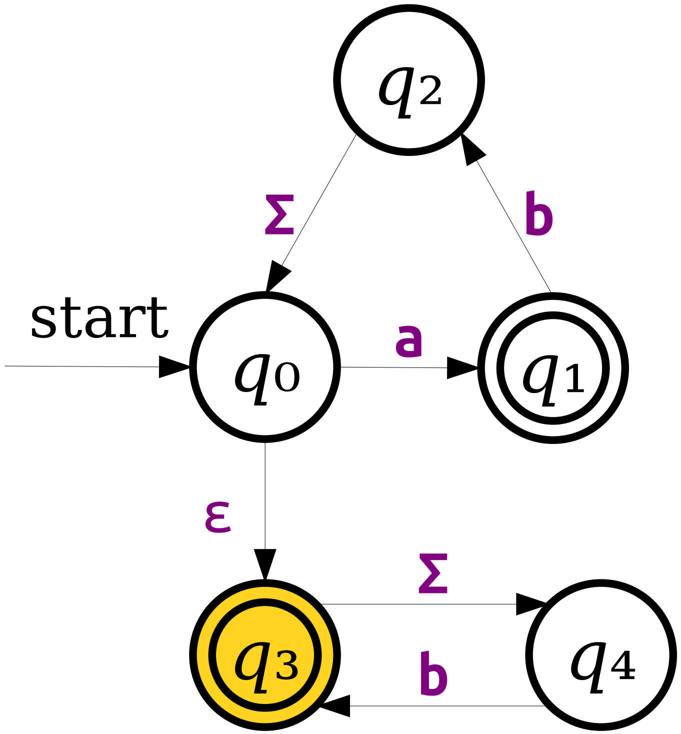
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	

… giving us this set of one active state…

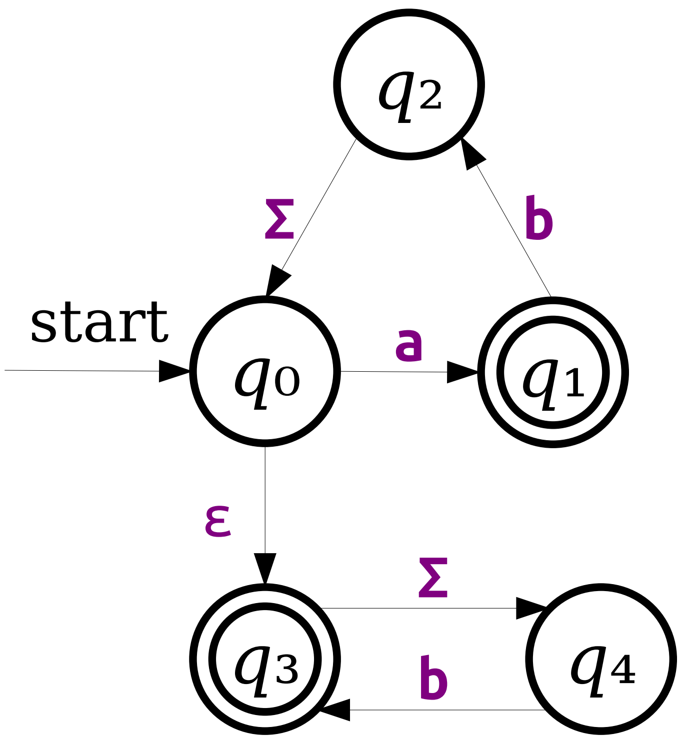
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

So we write down a singleton set for our result.

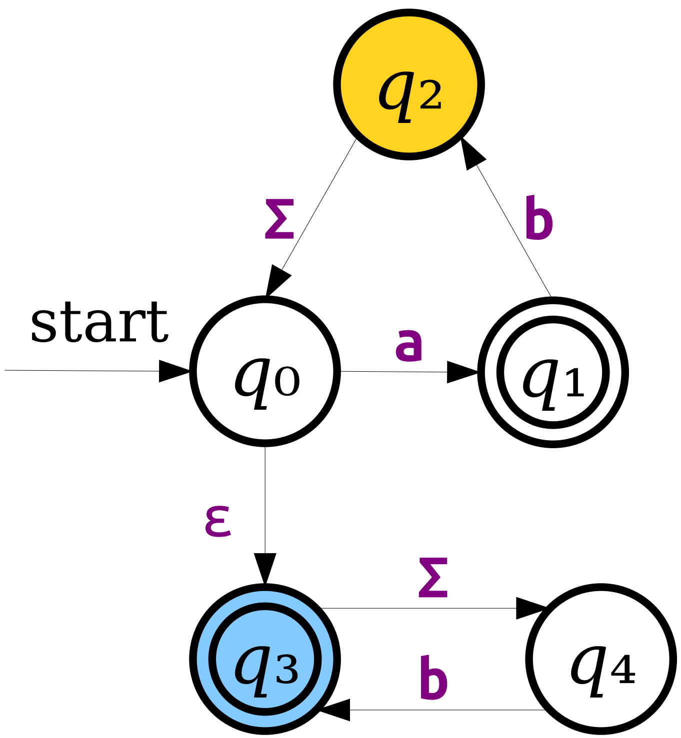
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$

This row is now complete. On to the next.

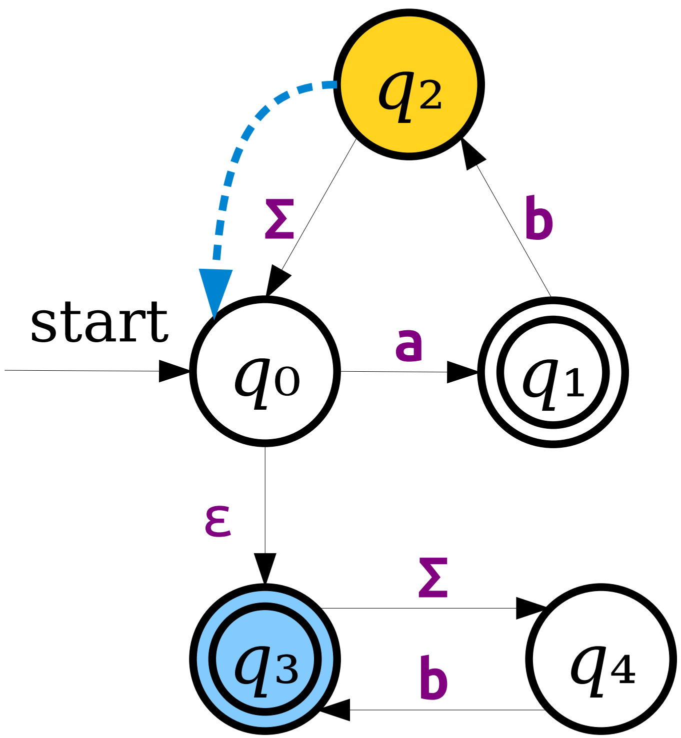
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

Things will get interesting now. A quick little time-saving observation: notice that the only transitions out of these states are Σ-transitions. That means the behavior will be the same on all possible characters. So where do we go if we read something?

Once More, With Epsilons!

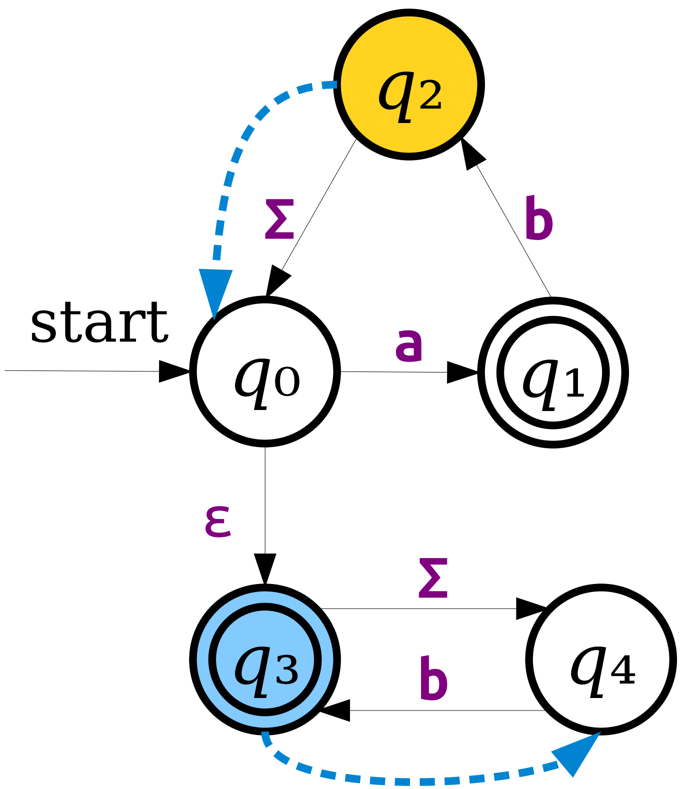


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

State q_2 goes to q_0.

(There's an ε-transition leaving state q_0 that we'll need to consider, but for now, let's ignore that.)

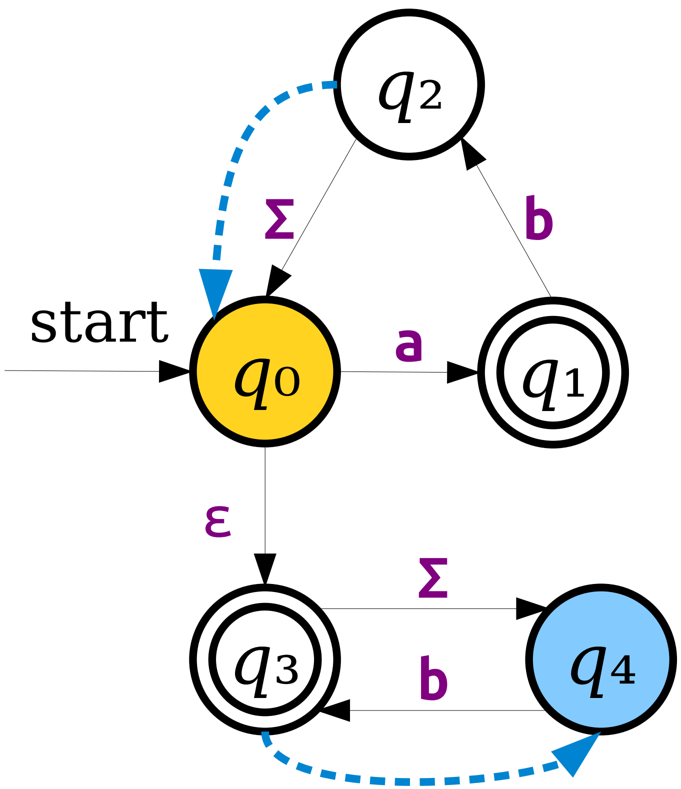
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

State q_3 goes to q_4.

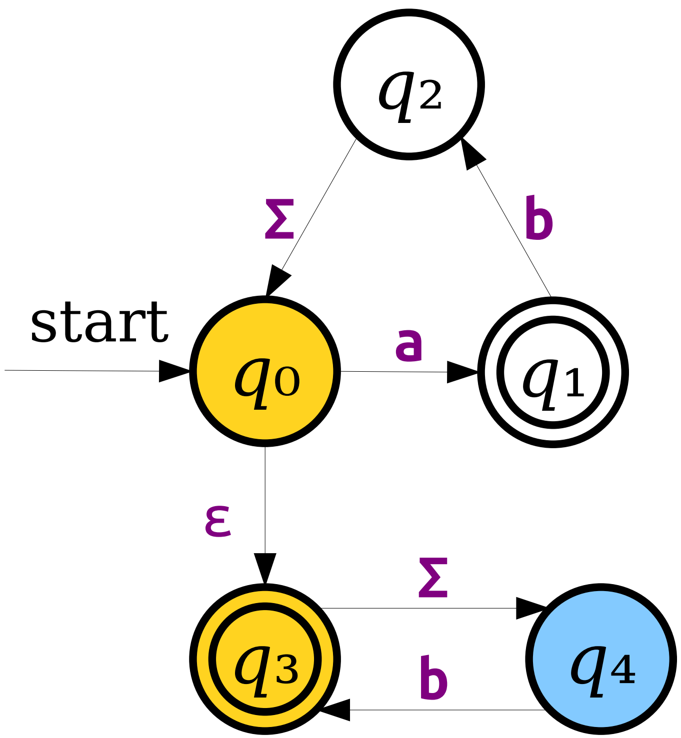
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

So that takes us here. Normally, we'd stop and write this combination down, but we aren't done yet. Importantly, there's an ε-transition leaving state q_0, and we have to consider that.

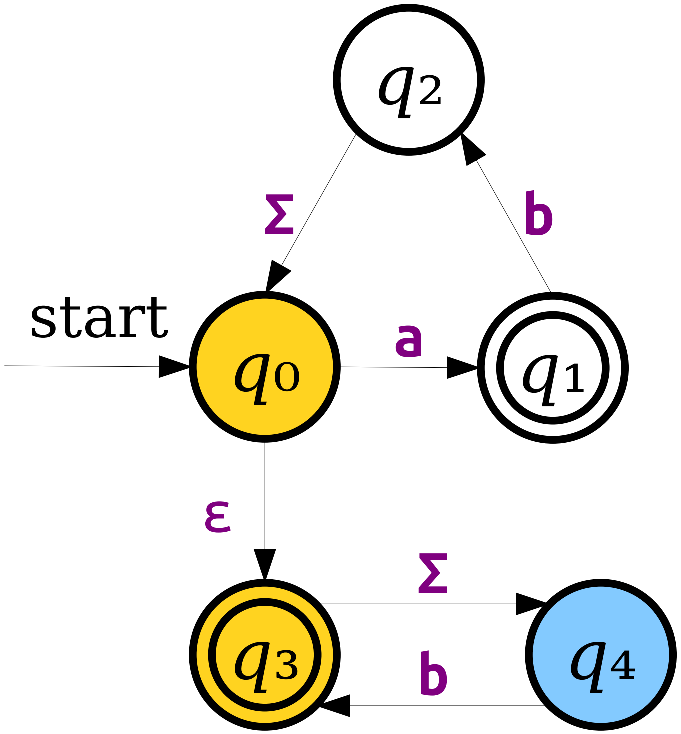
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$		

We'll therefore expand our set of states to include q_3. We're only now considering ε-transitions. Our policy will be to only care about ε-transitions in two cases: first, when determining the start state; second, *after* considering all transitions we can take by reading characters.

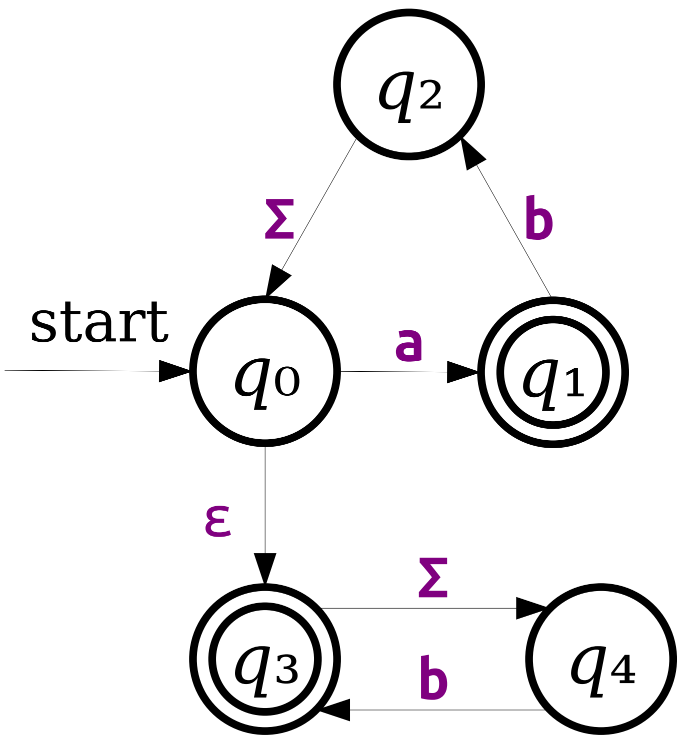
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

So we'll fill this set of states into our table for both a and b, since, as we saw earlier, the transitions out of q_2 and q_3 are Σ-transitions and thus work for all characters.

Once More, With Epsilons!

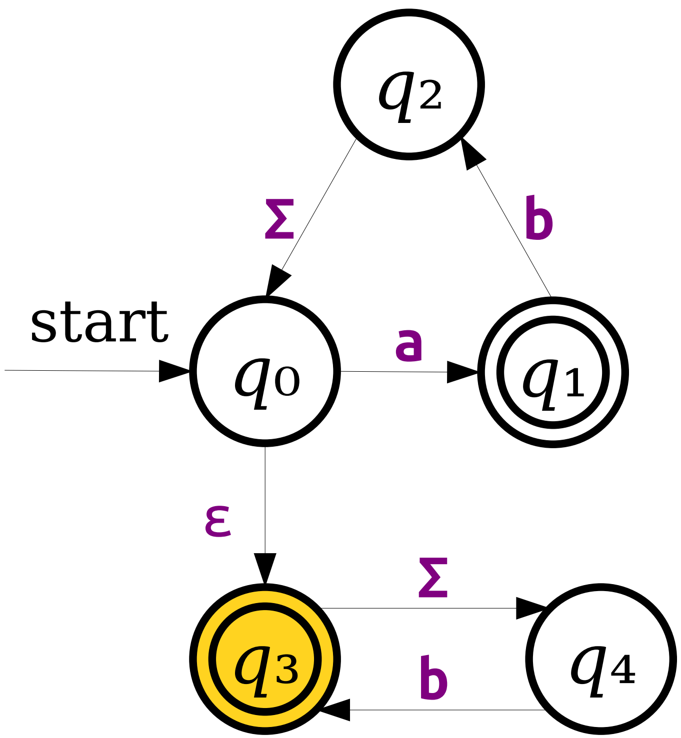


	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$

That's convenient, because we're now done with this row!

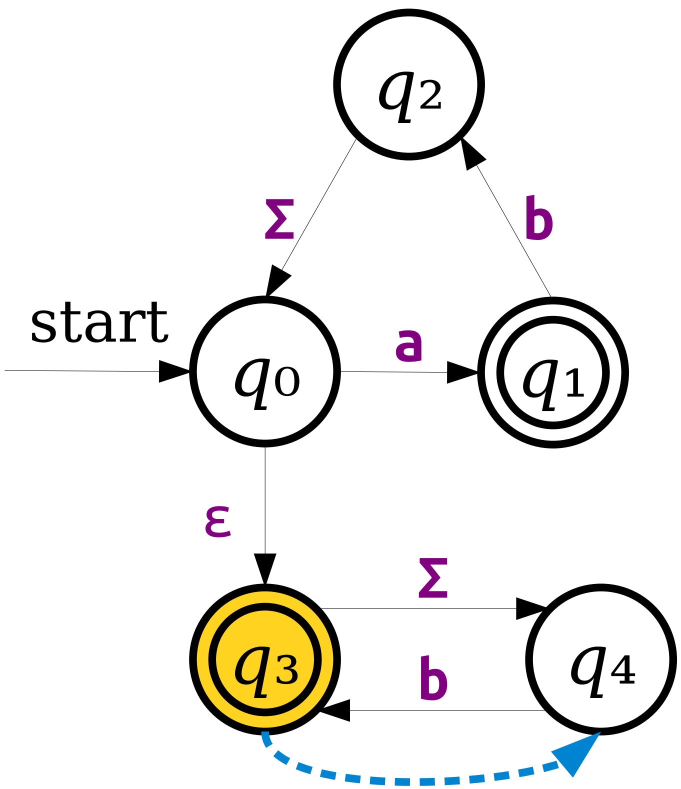
From here on out, we just keep applying these same rules. I'm going to pause on the narration until ***Slide 85***. Feel free to go one step at a time until then if you want to see the algorithm at work.

Once More, With Epsilons!



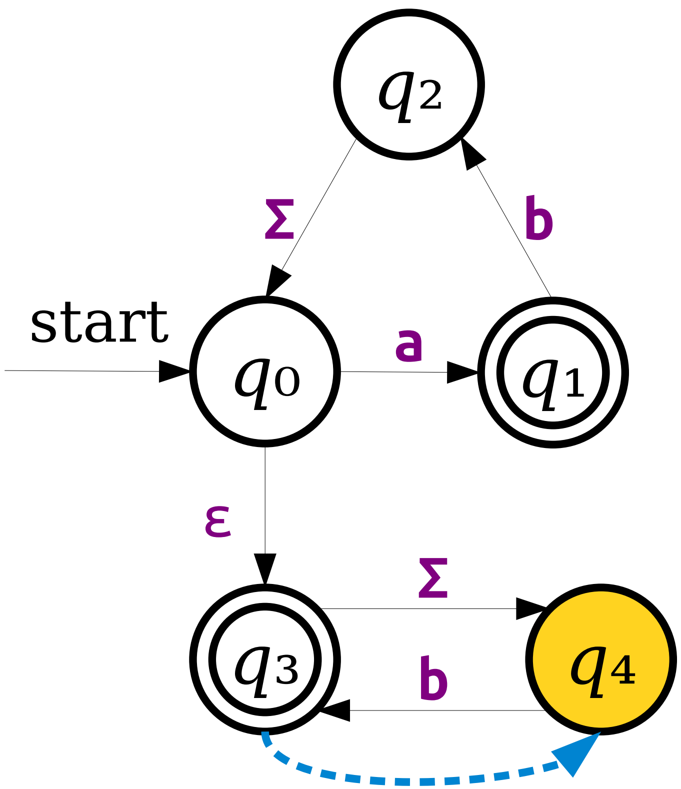
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



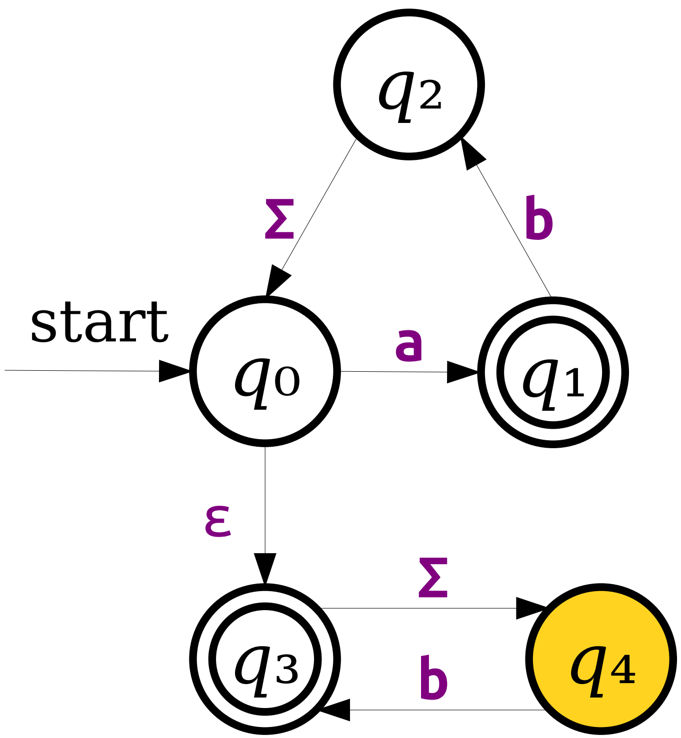
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



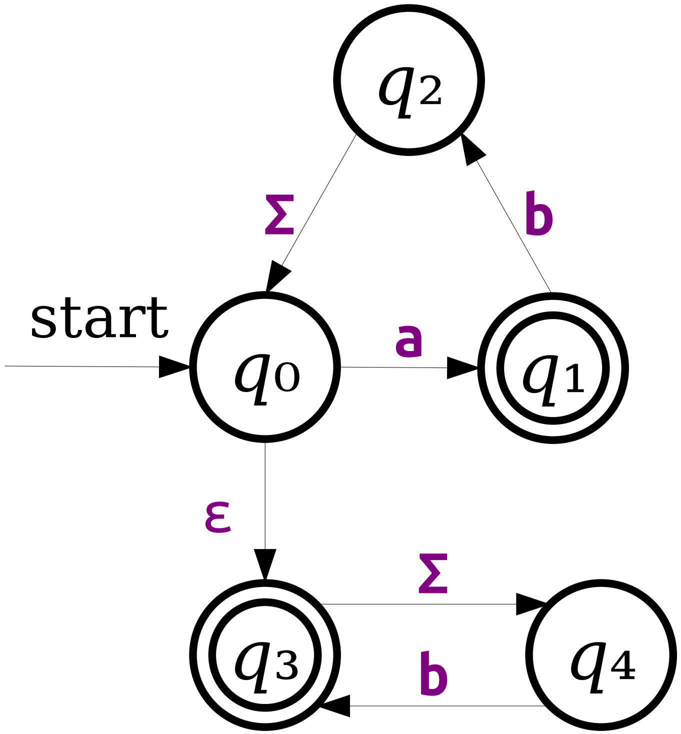
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$		

Once More, With Epsilons!



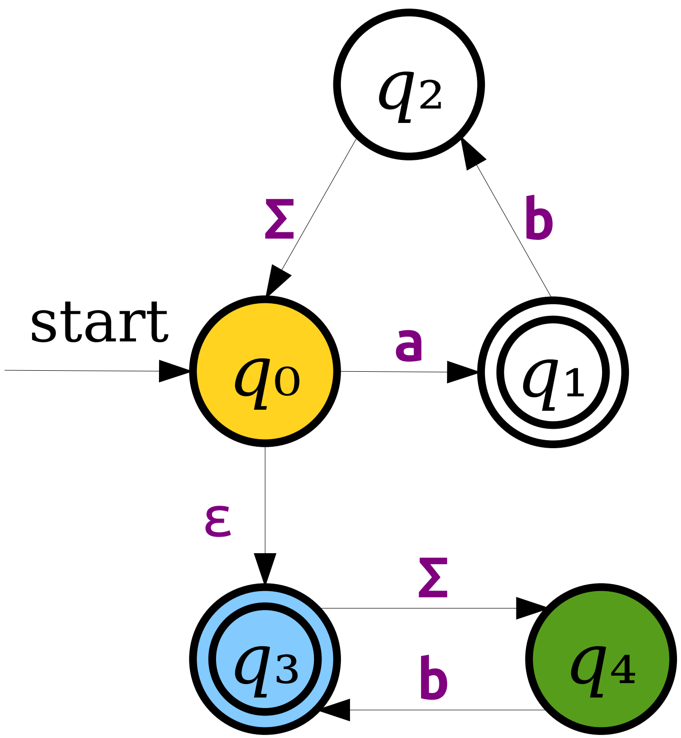
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



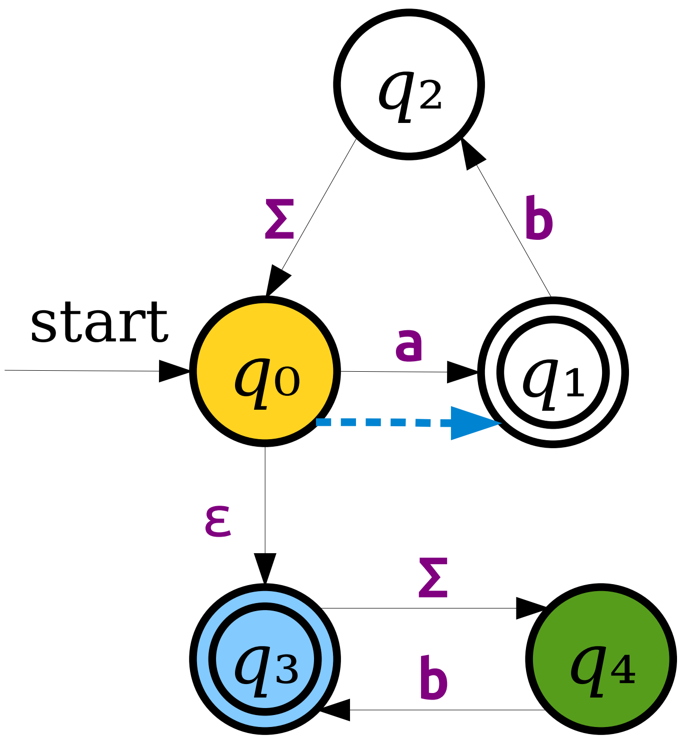
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$

Once More, With Epsilons!



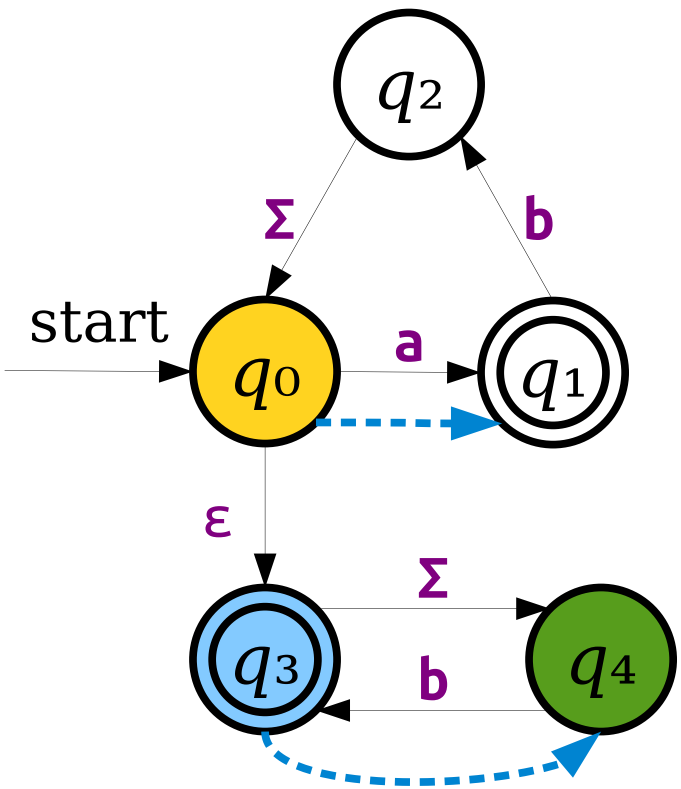
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



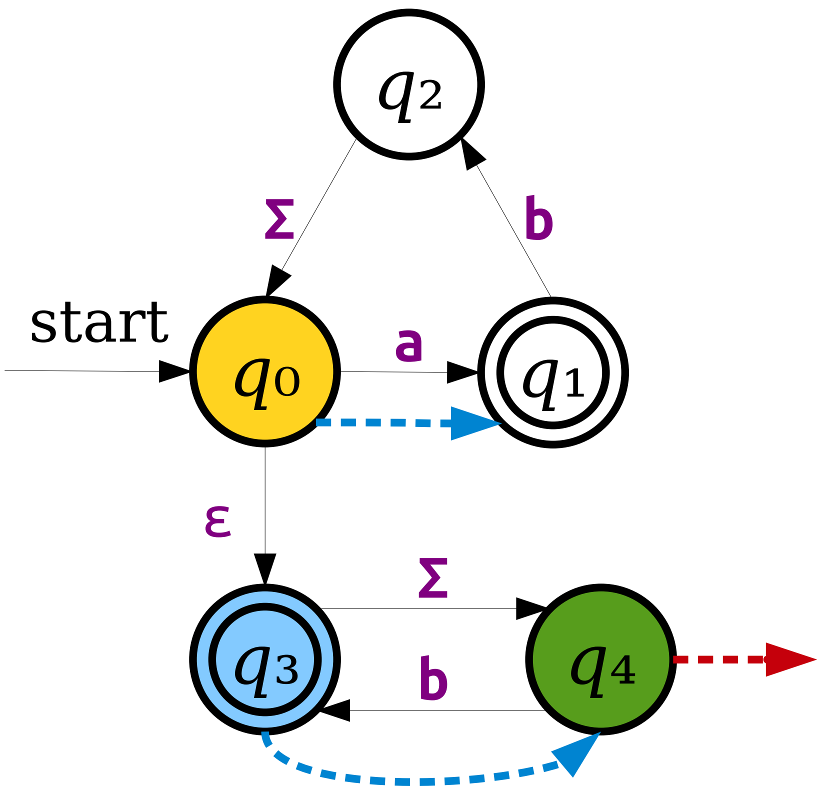
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



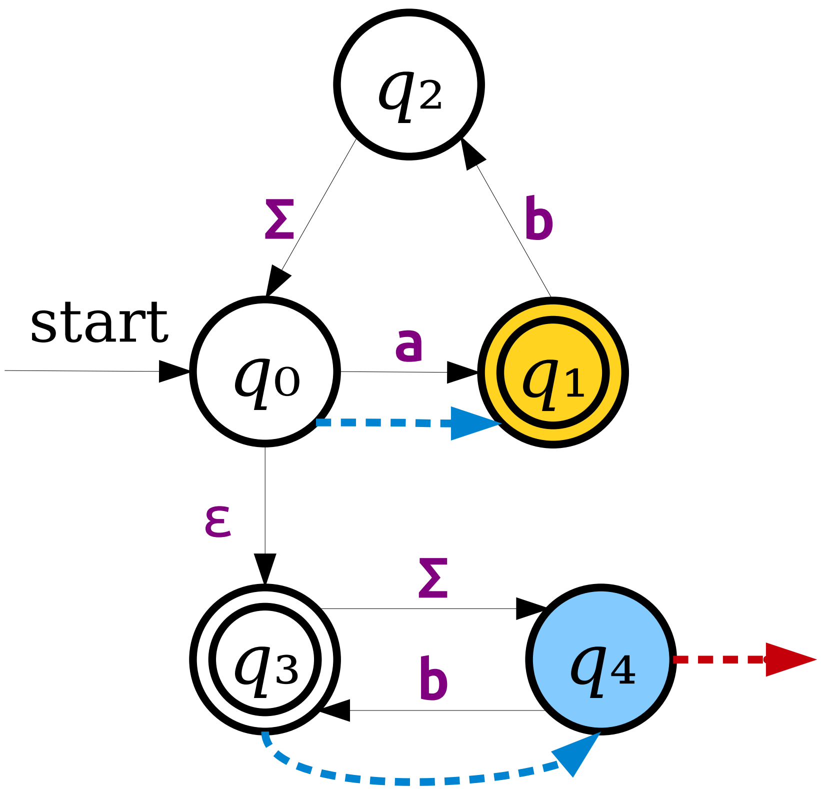
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



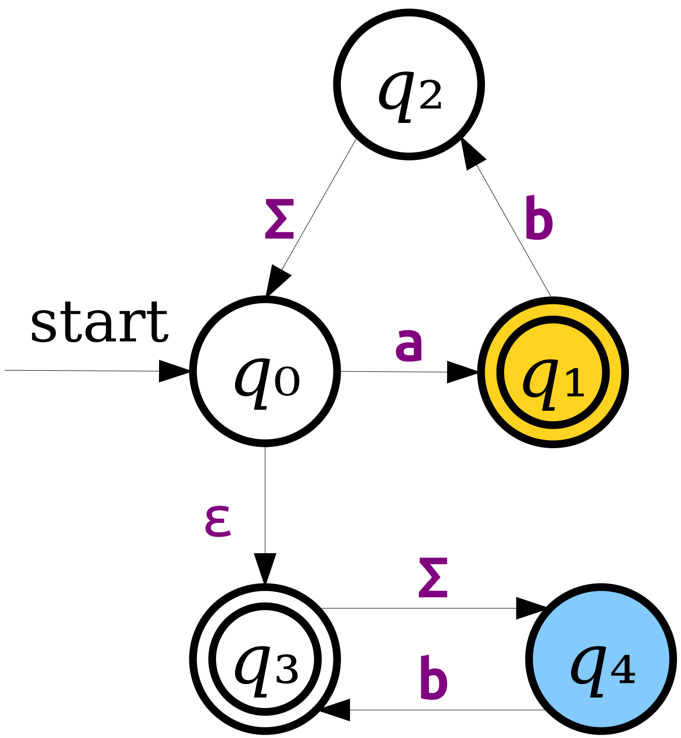
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



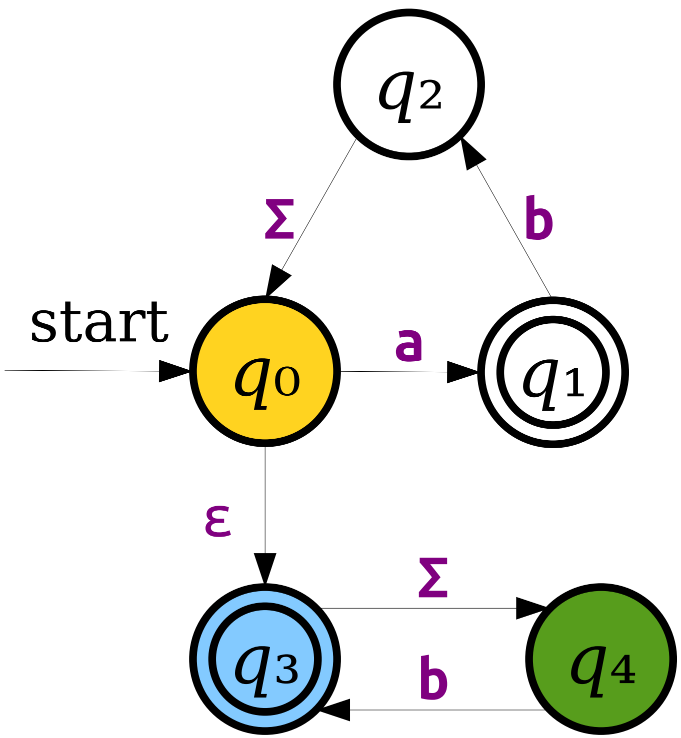
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$		

Once More, With Epsilons!



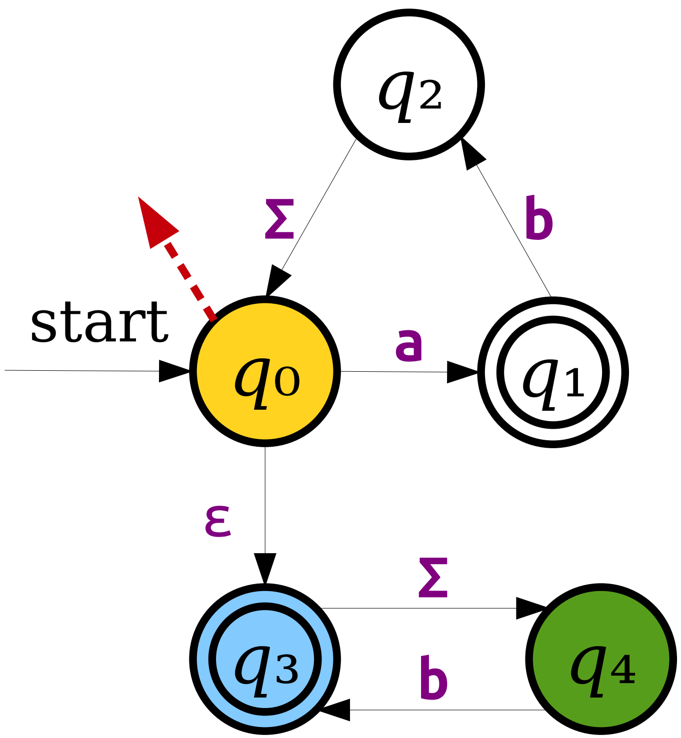
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



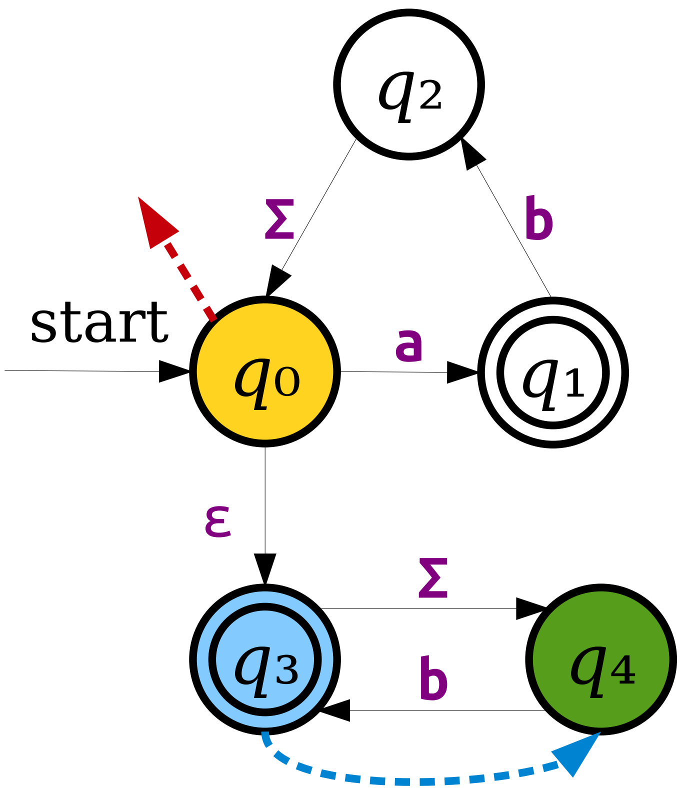
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



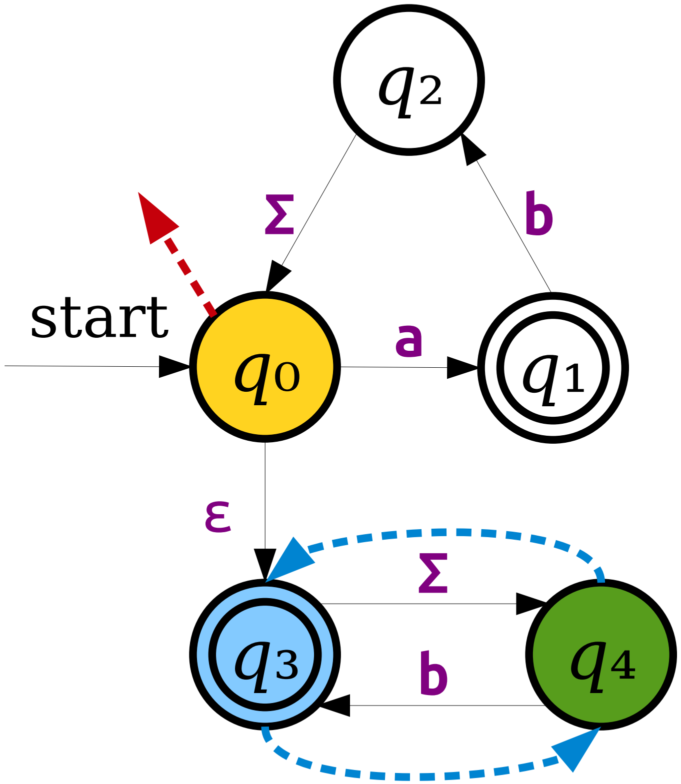
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



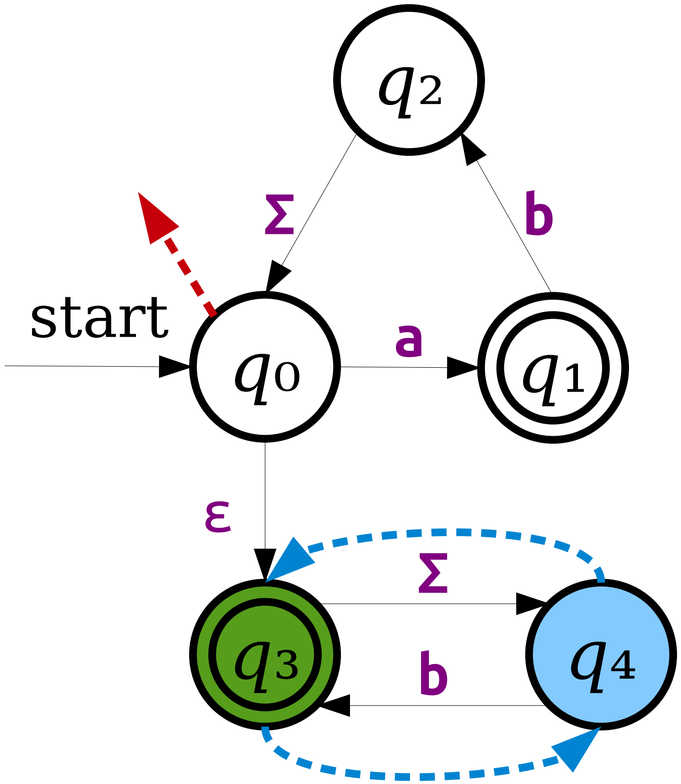
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



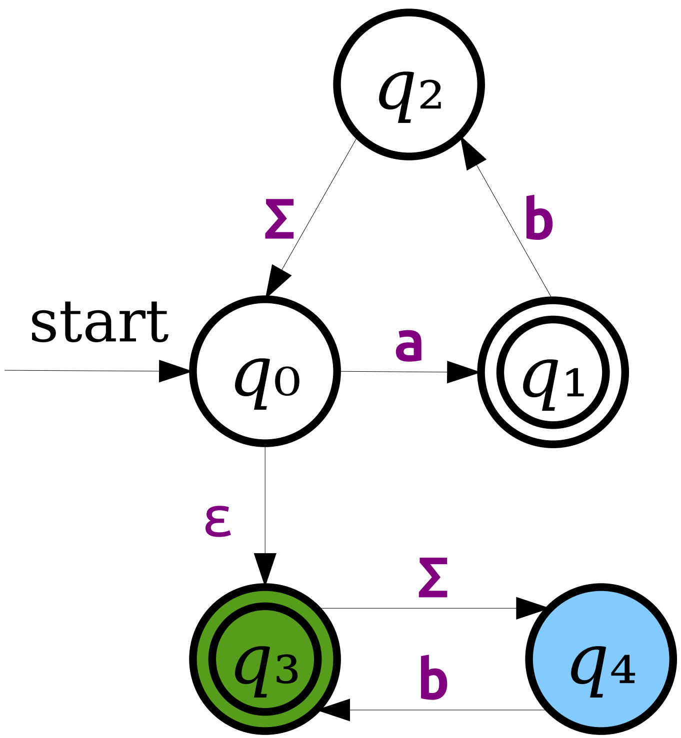
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



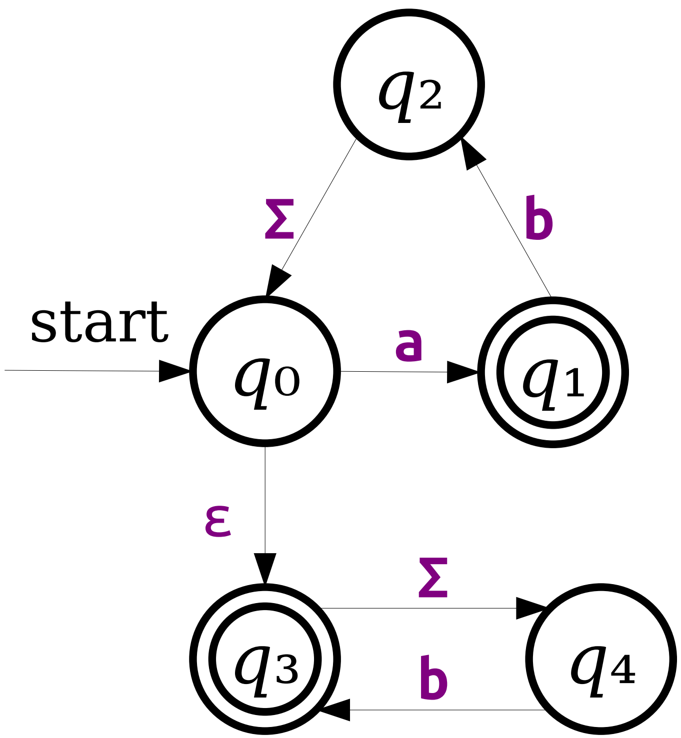
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	

Once More, With Epsilons!



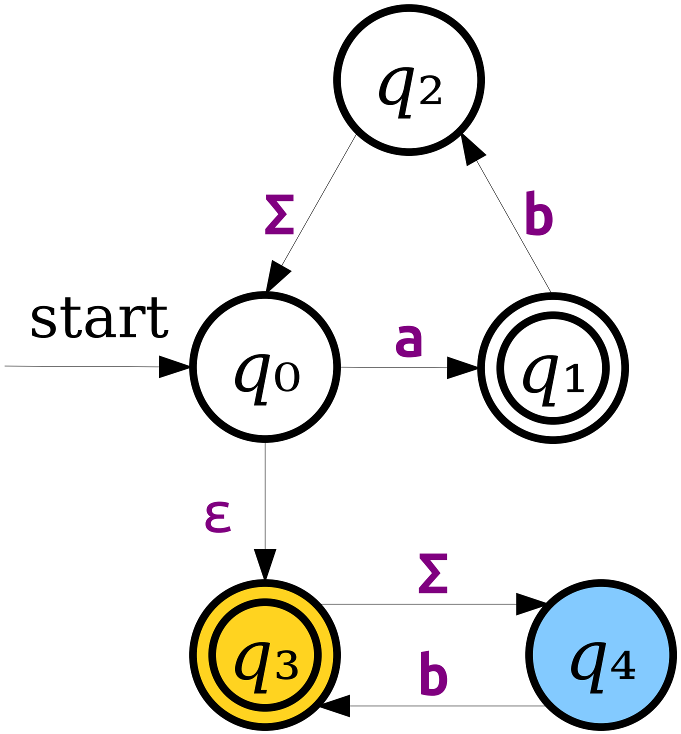
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



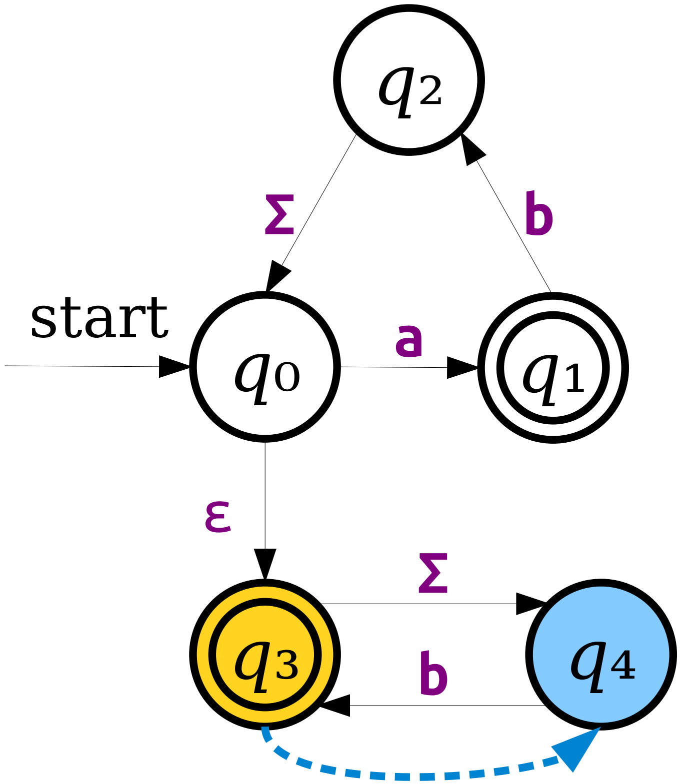
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



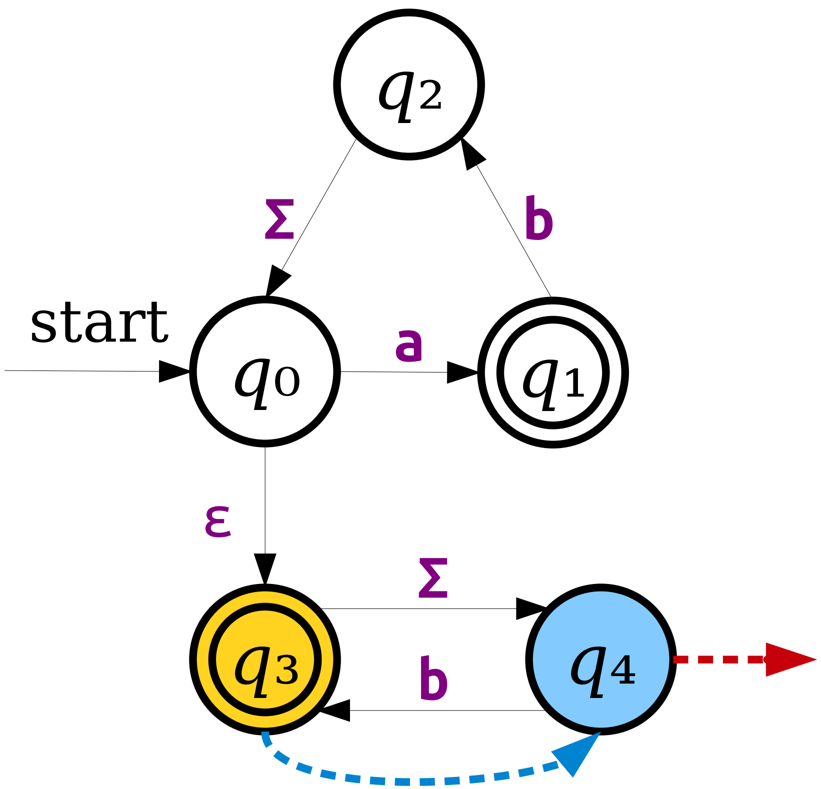
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



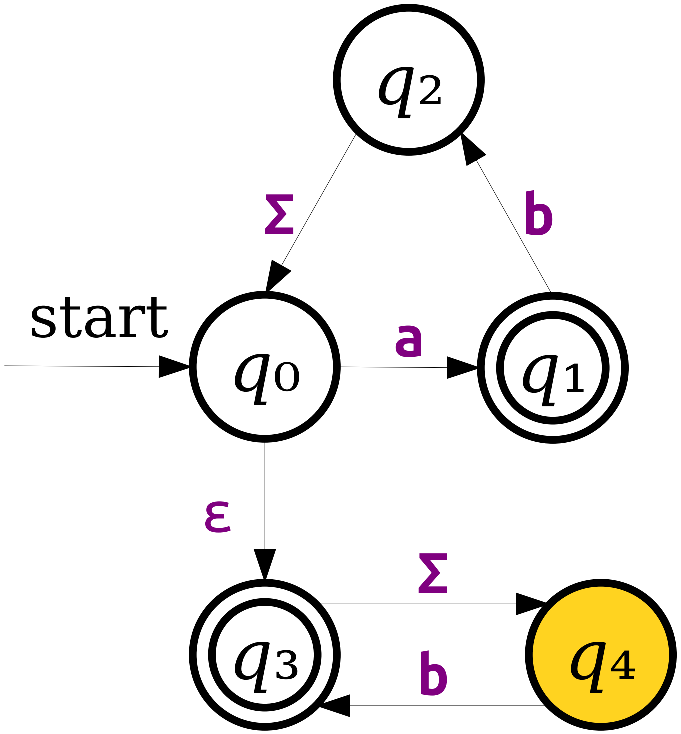
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



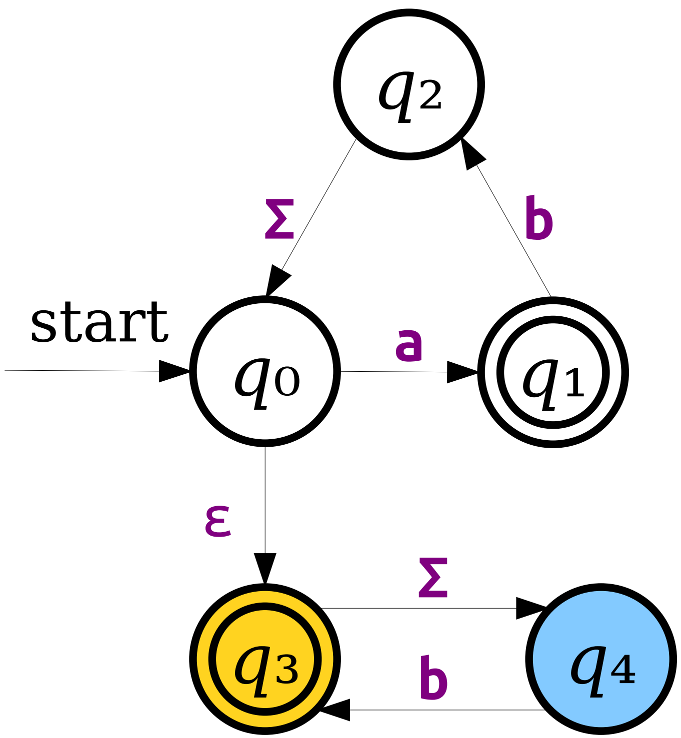
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$		

Once More, With Epsilons!



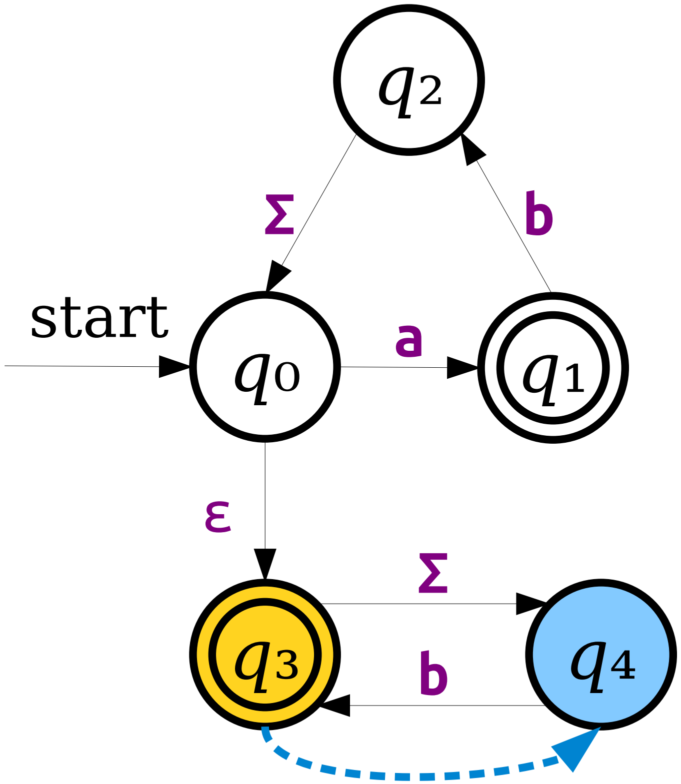
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



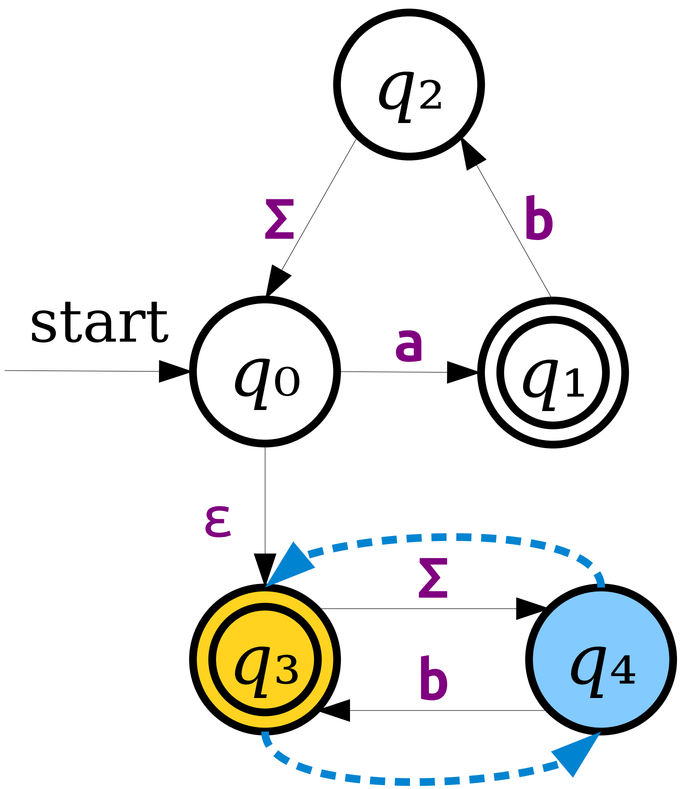
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



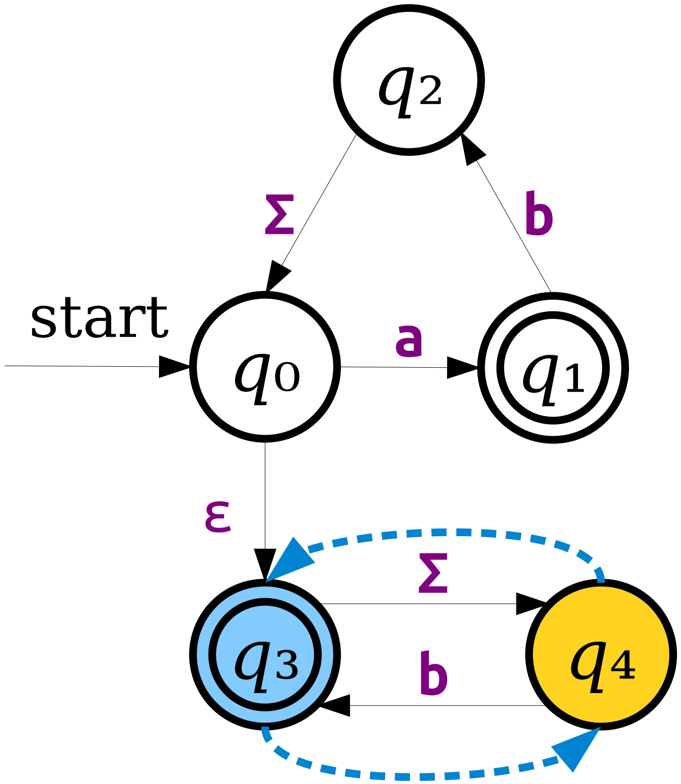
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



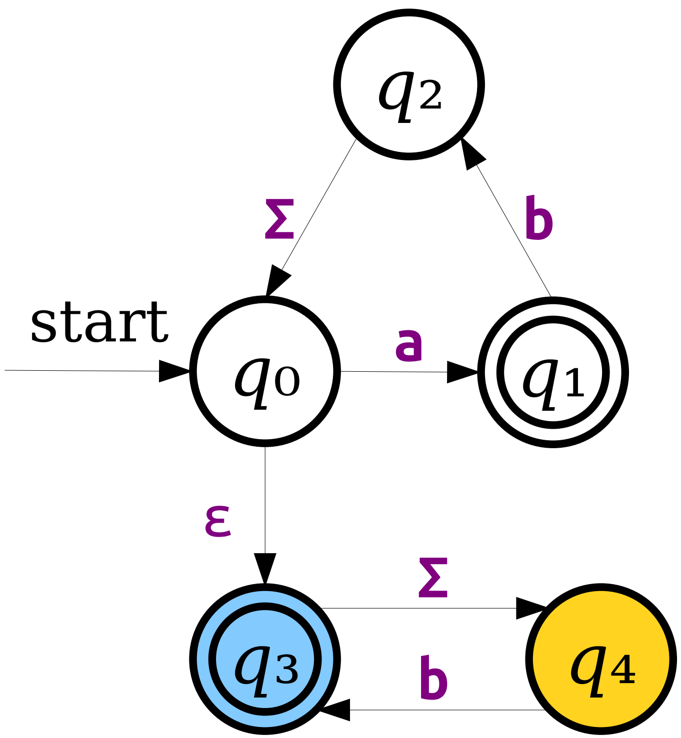
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



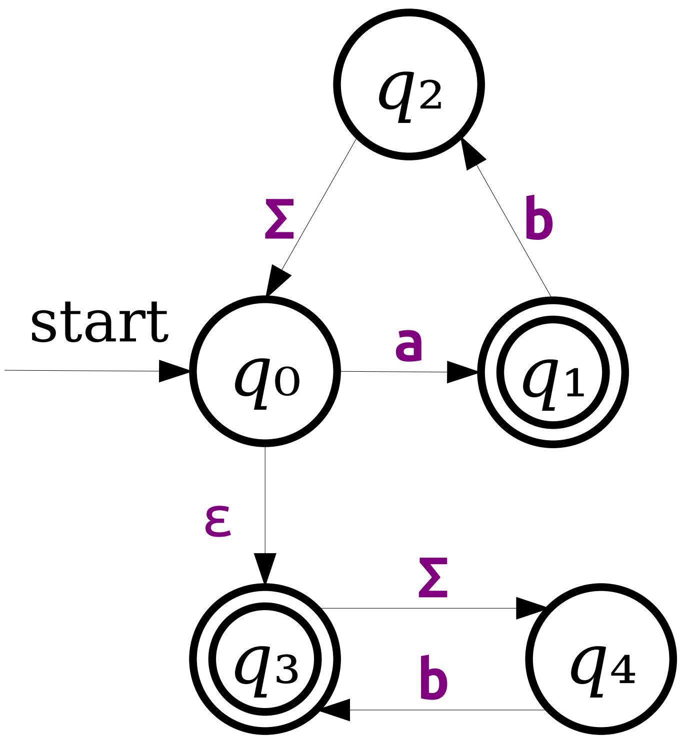
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	

Once More, With Epsilons!



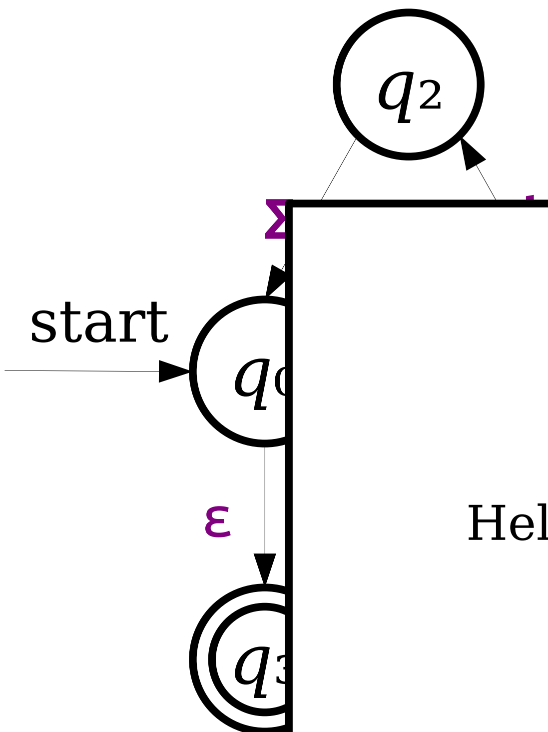
	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$

Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$

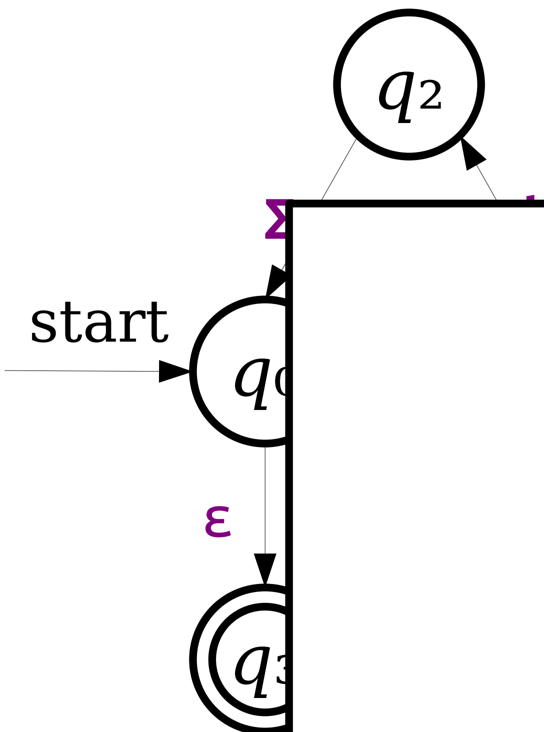
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
		$, q_3\}$
		$q_3\}$
		$q_3, q_4\}$
		$q_4\}$
		$, q_4\}$
		$, q_4\}$

Hello again! Time for the narration to resume.

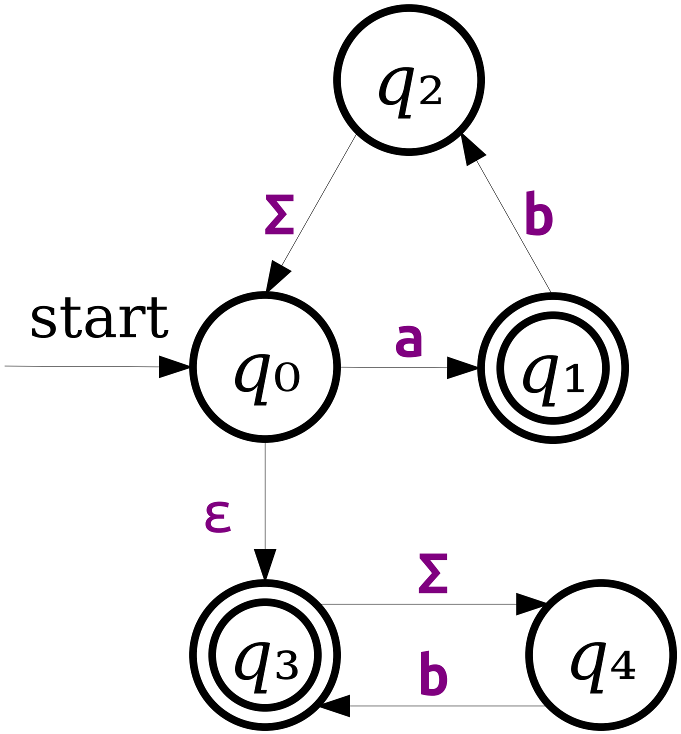
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
		$, q_3\}$
		$q_3\}$
		$q_3, q_4\}$
		$q_4\}$
		$, q_4\}$
		$, q_4\}$

Now that we have your attention...

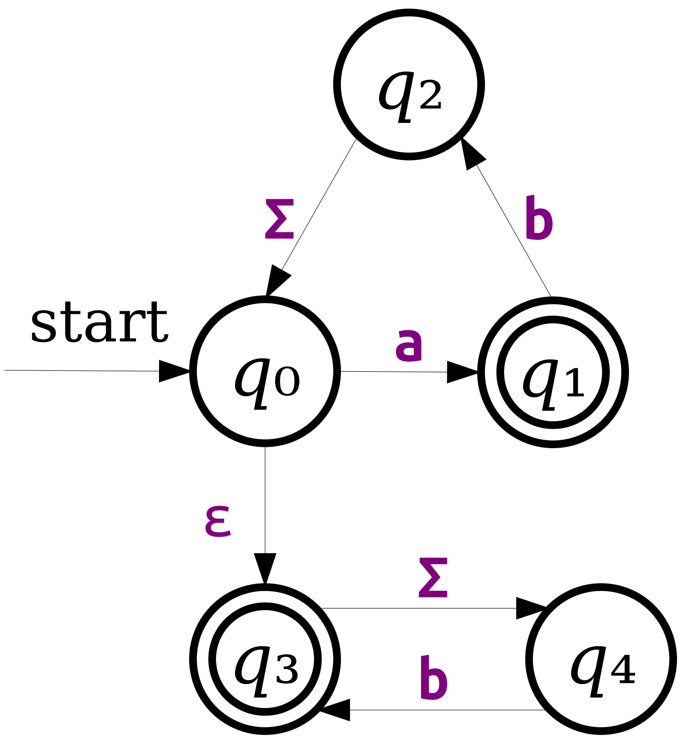
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$

We are almost done filling in this table. There's one set of states that's missing from here. Can you spot what it is?

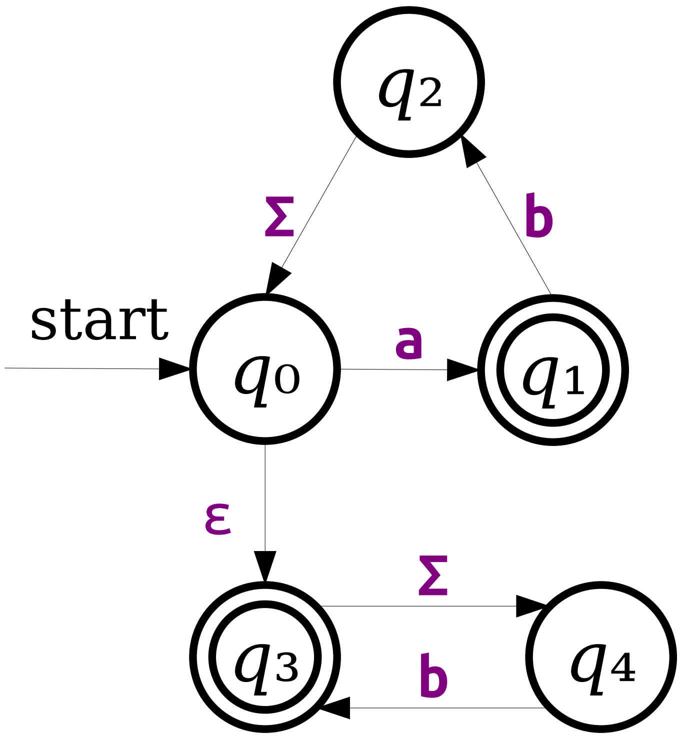
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$		

It's the empty set. What do we do here?

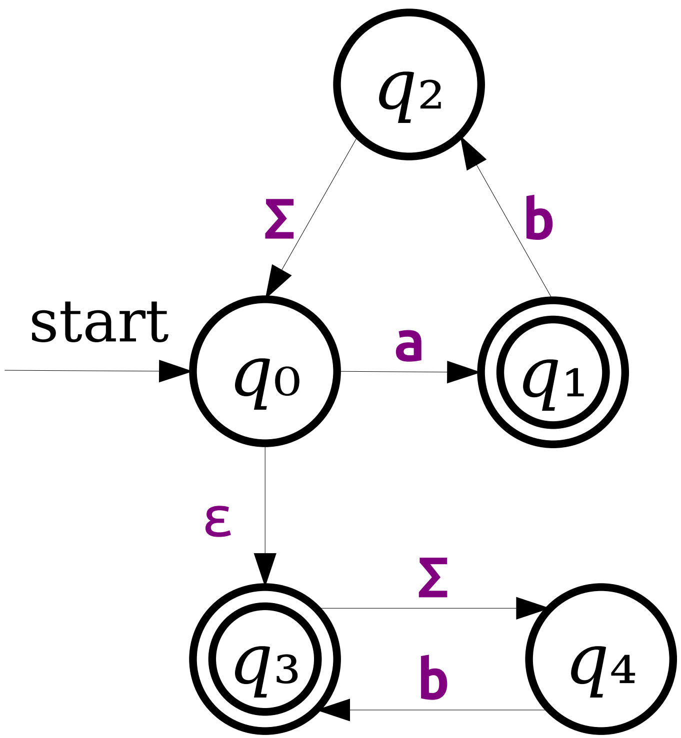
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
Ø		

Well, the rule is to look at what happens if those states are active and to ask what the NFA would do in that configuration.

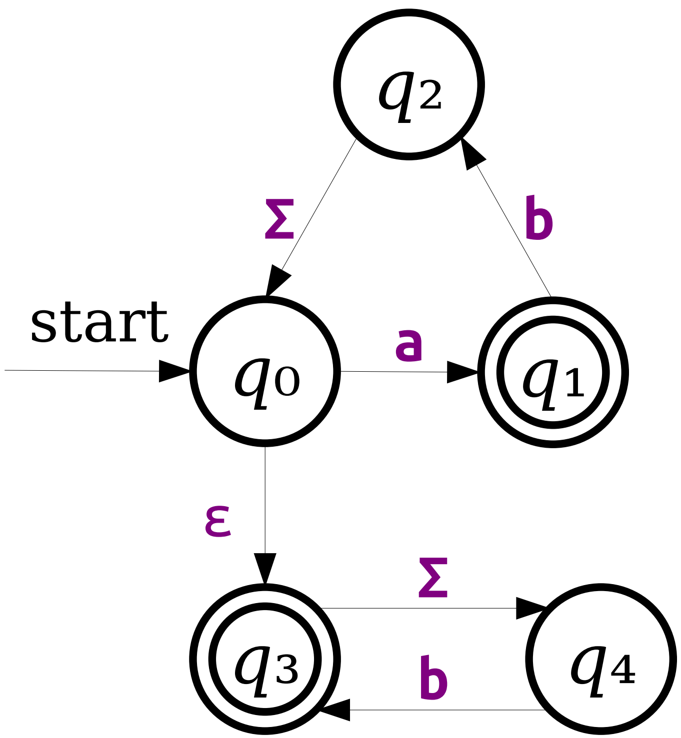
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
Ø		

In this case, if the NFA is not in any states, the NFA has died off. And reading any characters isn't going to change that! The NFA is still dead.

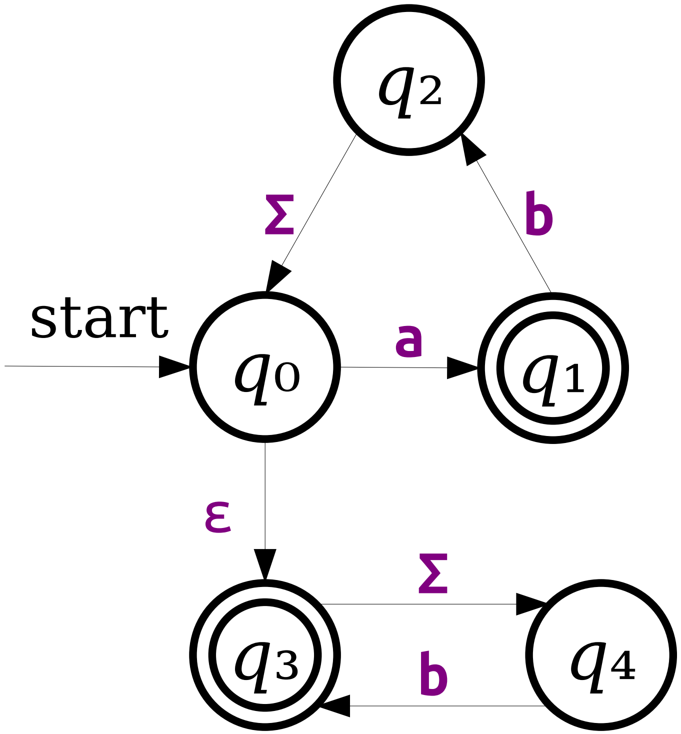
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$		

So, if we start in set of states Ø, we stay in that set of states regardless of what we read. And that's how we'll fill the table in.

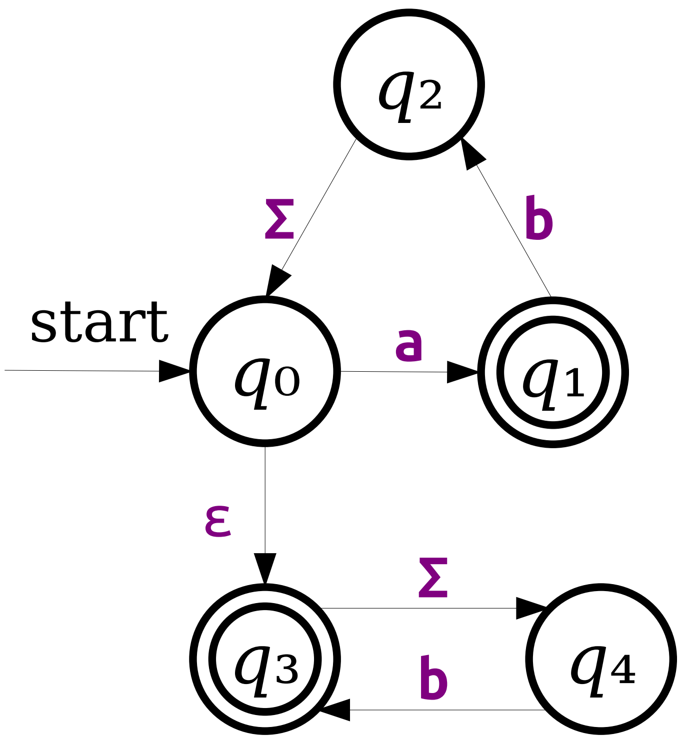
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
Ø	Ø	Ø

We'll put Ø in both columns. This really is a dead state – once we're here, there's no turning back! We can never leave.

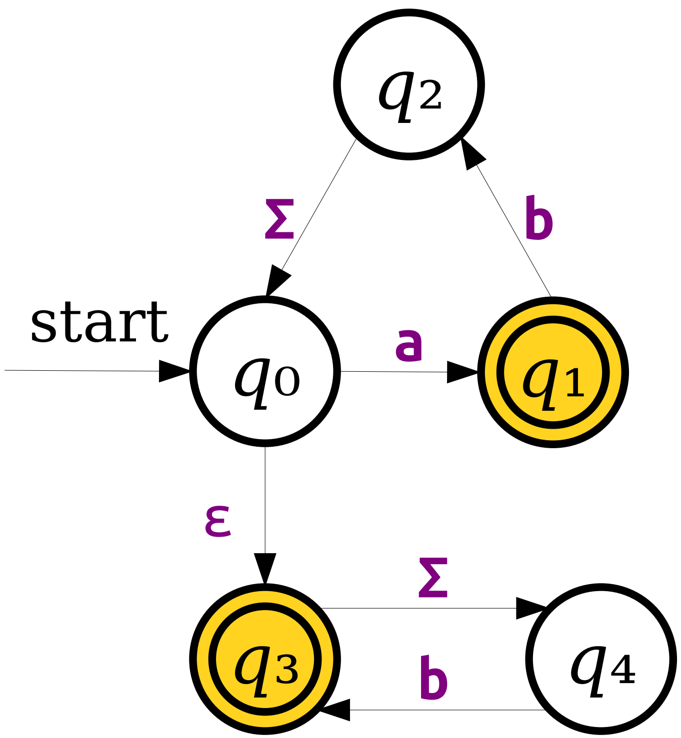
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\varnothing$	$\{q_2, q_3\}$
$\{q_4\}$	$\varnothing$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\varnothing$	$\varnothing$	$\varnothing$

We've just about finished doing the subset construction. We now have all the DFA states and transitions. But something's missing… accepting states!

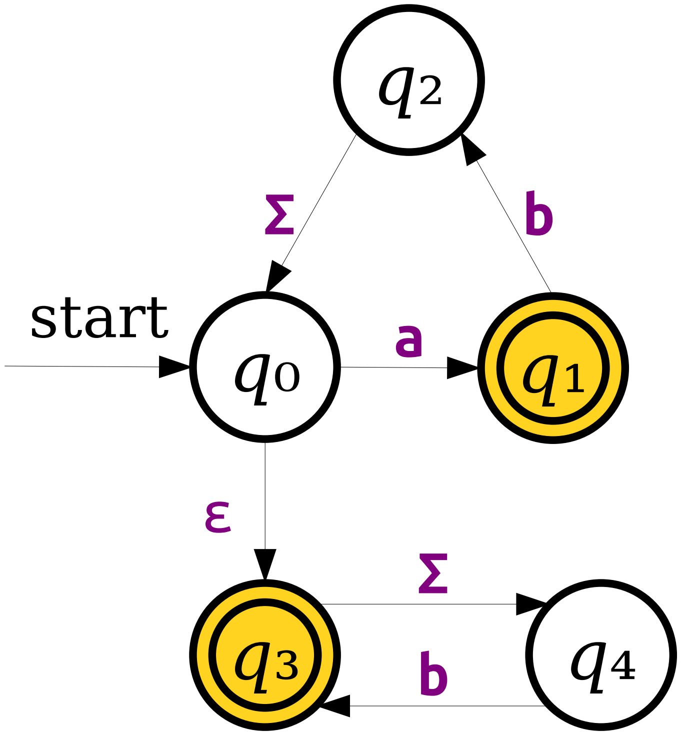
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
		$\{q_4\}$
		$\{q_3, q_4\}$
		$\{q_3, q_4\}$
		$\emptyset$

As a reminder, the rule is to look at the states in the NFA that are accepting and to mark each DFA state as accepting if it contains at least one NFA accepting state. The idea here is that if the NFA ends up in a combination of states with an accepting state, we accept, so the DFA has to simulate that.

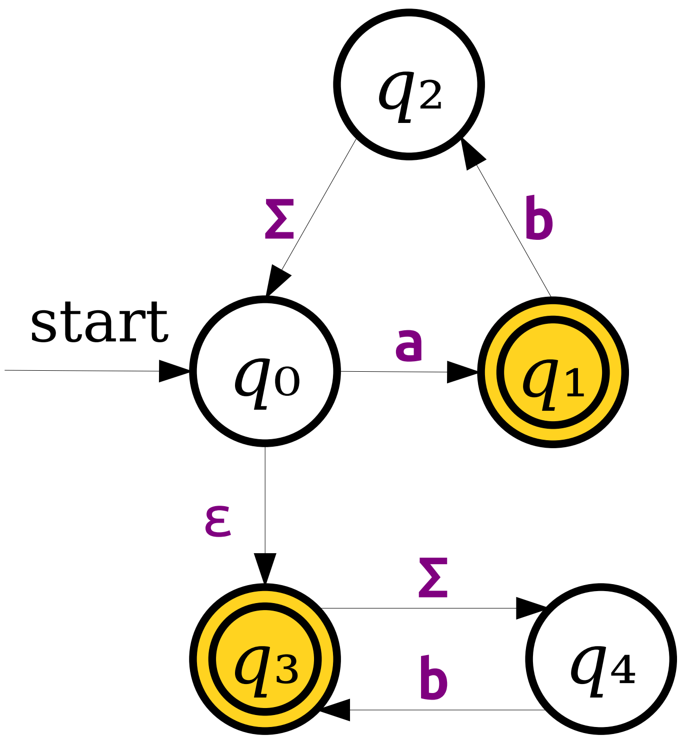
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	$\emptyset$	$\{q_2, q_3\}$
$\{q_4\}$	$\emptyset$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
$\emptyset$	$\emptyset$	$\emptyset$

Based on that, which states should be accepting? Make a guess, then move on.

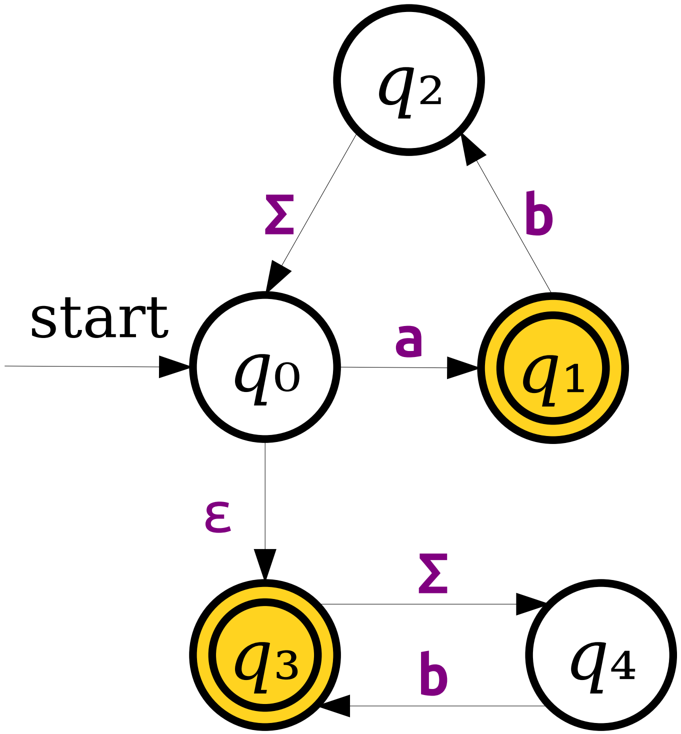
Once More, With Epsilons!



	a	b
$\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
Ø	Ø	Ø

You've really made a guess?
Great!

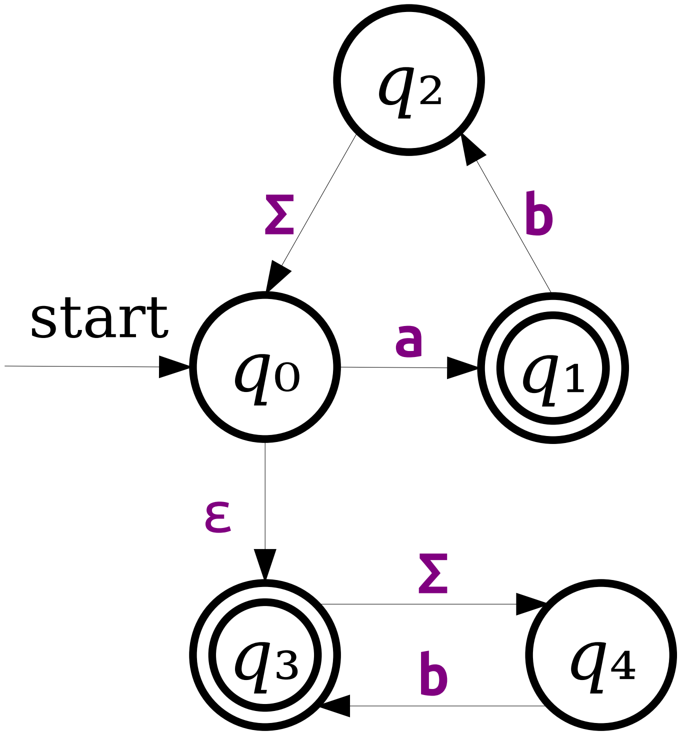
Once More, With Epsilons!



	a	b
$*\{q_0, q_3\}$	$\{q_1, q_4\}$	$\{q_4\}$
$*\{q_1, q_4\}$	Ø	$\{q_2, q_3\}$
$\{q_4\}$	Ø	$\{q_3\}$
$*\{q_2, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_3, q_4\}$
$*\{q_3\}$	$\{q_4\}$	$\{q_4\}$
$*\{q_0, q_3, q_4\}$	$\{q_1, q_4\}$	$\{q_3, q_4\}$
$*\{q_3, q_4\}$	$\{q_4\}$	$\{q_3, q_4\}$
Ø	Ø	Ø

Here's the answer. Most of these states are accepting!

Once More, With Epsilons!



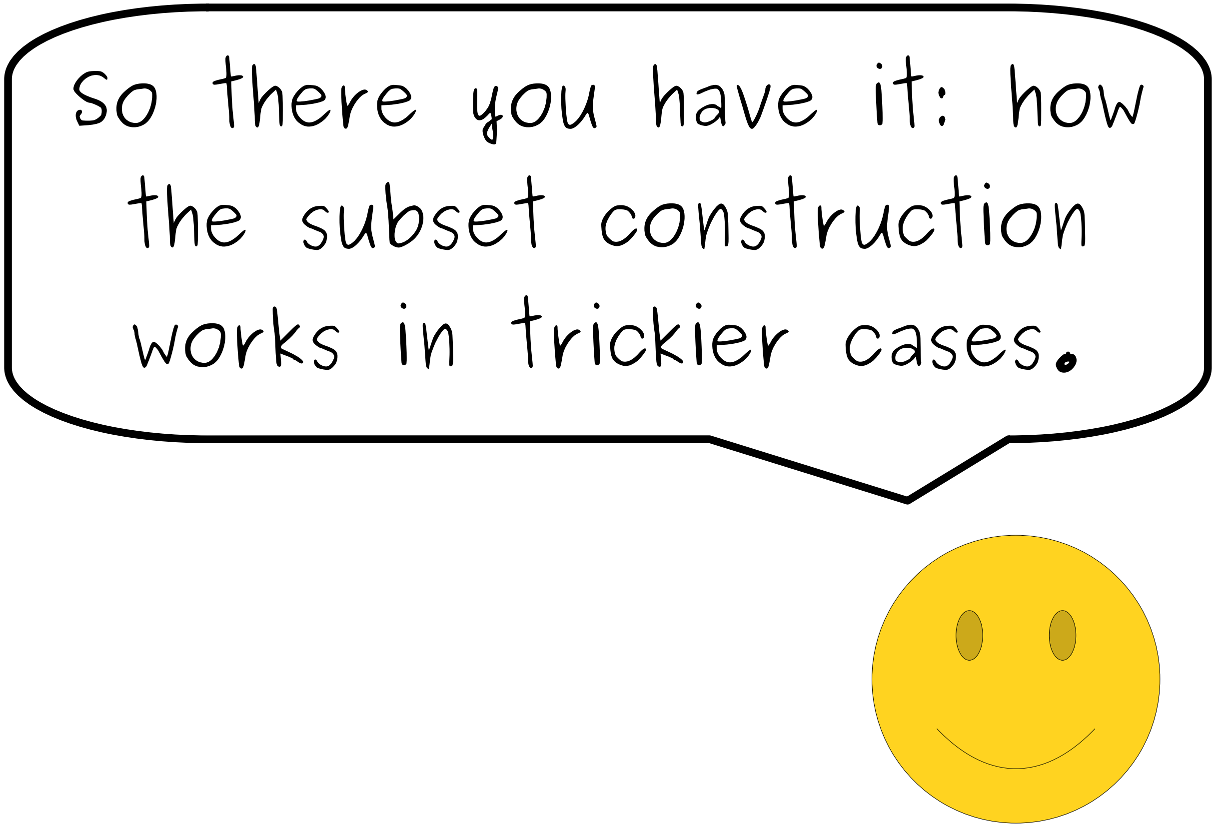
	a	b
*{q_0, q_3}	{q_1, q_4}	{q_4}
*{q_1, q_4}	Ø	{q_2, q_3}
{q_4}	Ø	{q_3}
*{q_2, q_3}	{q_0, q_3, q_4}	{q_0, q_3, q_4}
*{q_3}	{q_4}	{q_4}
*{q_0, q_3, q_4}	{q_1, q_4}	{q_3, q_4}
*{q_3, q_4}	{q_4}	{q_3, q_4}
Ø	Ø	Ø

And that's it, we're done!

The next slide just has a summary of the subset construction on it. It’s really formal and is designed to combine together everything we’ve talked about so far. If you need to code up the subset construction for some reason, the next slide is a great reference! If not, think of it as a way of deciding what to do in super tricky edge cases.

The Subset Construction

- Each state in the DFA is associated with a set of states in the NFA.
- The start state in the DFA corresponds to the start state of the NFA, plus all states reachable via ε-transitions.
- If a state *q* in the DFA corresponds to a set of states *S* in the NFA, then the transition from state *q* on a character `a` is found as follows:
  - Let *S*' be the set of states in the NFA that can be reached by following a transition labeled `a` from any of the states in *S*. *(This set may be empty.)*
  - Let *S*'' be the set of states in the NFA reachable from some state in *S*' by following zero or more epsilon transitions.
  - The state *q* in the DFA transitions on `a` to a DFA state corresponding to the set of states *S*''.


So there you have it: how the subset construction works in trickier cases.

Did you find this useful?
If so, let us know and
we can make more guides
like these for other topics.